L'ALGEBRA
OPERA

Di RAFAEL BOMBELLI da Bologna
Diuiſa in tre Libri.

Con la quale ciaſcuno da ſe potrà venire in perfetta cognitione della teorica dell' Arimetica.

Con vna Tauola copioſa delle materie, che in eſſa ſi contengono.

Poſta hora in luce à beneficio delli Studioſi di detta profeſſione.

IN BOLOGNA,
Per Giouanni Roſsi. MDLXXIX.
Con licenza de' Superiori (1560)

A rework of Cardano's Ars Magna

Rafael Bombèlli
(1526-1572)

$x^3 = 15x + 4$

Cardano's Formula gives solution

$x = \sqrt[3]{2+A} + \sqrt{2-A}$, $A^2 = -121$

Bombelli's technique "rests on sophistry rather than in truth"

write $\sqrt[3]{2+A}$ as $a+\sqrt{-b}$
$\sqrt[3]{2-A}$ as $a-\sqrt{-b}$

$a^2+b = \sqrt[3]{(2+A)(2-A)}$
$= \sqrt[3]{2+121} = 5$

$a^3 - 3ab = 2$
$a^2 + b = 5$

Bombelli noted an integer solution
$b=1$, $a=2$ $x = (2+i)+(2-i) = 4$
(There is a second solution $a = -1.866$
$b + 1.518$
solutions; -0.268, 4, -3.732

François Viète

(1540-1603)

François Viète began with a career as a lawyer in his home town in west-central France.

He rose to a counsellor's position in Brittany before being forced out of office for being not Roman Catholic (a Huguenot). In 1588, when Henry IV became king, he was reinstated as a counsellor in Tours.

While at Tours he discovered the key to a Spanish cipher, which helped Henry IV in his military campaigns. Afterwards, Viete became a royal privy counsellor, and remained so till his death, which took place suddenly and suspiciously in Paris in February 1603.

Being wealthy, he printed numerous papers at his own expense. His text *Introduction to the Analytic Art*, or *Isagoge in artem analyticam in* 1591 was his major work.

Introduced letters for variables (partway to modern use)

First occurrence of modern quadratic formula.

Still did not consider negative (or complex) roots of equations.

Viete's Formula:

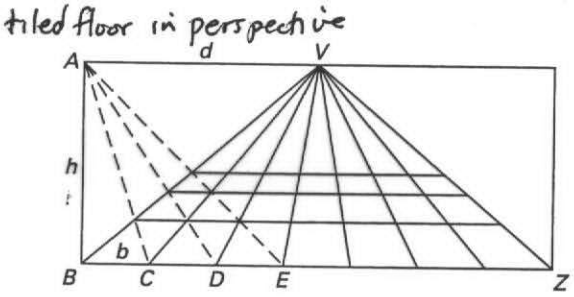
For any general polynomial of degree $n \geq 1$

$$p(X) = a_n X^n + a_{n-1} X^{n-1} + \cdots + a_1 X + a_0$$

$$\begin{cases} x_1 + x_2 + \cdots + x_{n-1} + x_n = \frac{-a_{n-1}}{a_n} \\ (x_1x_2 + x_1x_3 + \cdots + x_1x_n) + (x_2x_3 + x_2x_4 + \cdots + x_2x_n) + \cdots + x_{n-1}x_n = \frac{a_{n-2}}{a_n} \\ \vdots \\ x_1x_2 \ldots x_n = (-1)^n \frac{a_0}{a_n}. \end{cases}$$

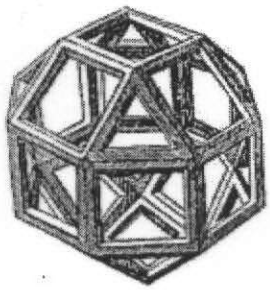
Perspectives in painting

Leon Battista Alberti (1404-1472)



Leonardo da Vinci (1452 – 1519)

Established all symmetry groups of the plane: cyclic groups (rotation) and the dihedral groups (the 2n symmetries of the n-gon).



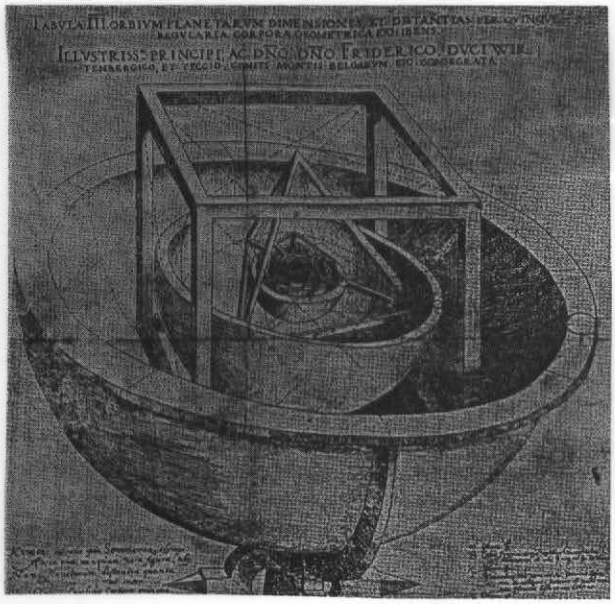
In the 1490s he studied mathematics under Luca Pacioli and prepared a series of drawings of regular solids in a skeletal form to be engraved as plates for Pacioli's book *De Divina Proportione*, published in 1509.

Johannes Kepler (1571-1630)

Born in the Black Forest (Southwest Germany). Despite his desire to become a minister, when he graduated from the local seminary Kepler was recommended for a position as teacher of mathematics and astronomy at the Protestant school in Graz, Austria (1594)

1595-1598: wrote *Mysterium Cosmographicum:* planets separated by Platonic solids

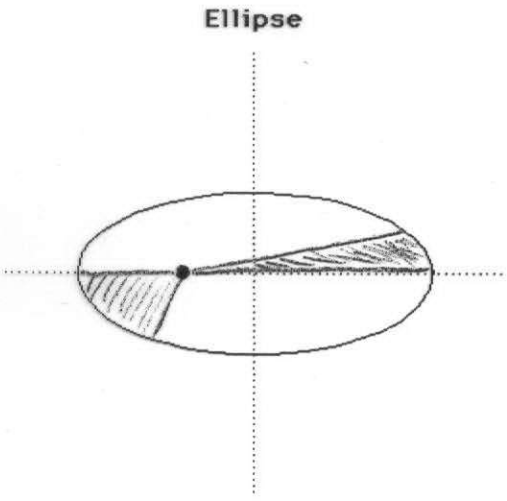
1600: Met Tycho Brahe, and started interpreting his observations of Mars as a graph.



1604: ***Kepler's Second Law: equal area in equal time***

Also realized that orbit was a kind of oval.

1609: After 40+ tries, figured out that Mars' orbit was an ellipse. Stated ***Kepler's First Law***: Planets orbit in elliptical orbits with Sun at one vertex.



Thomas Harriot (1560-1621)

Born in Oxford, after graduation he went to work for Sir Walter Raleigh as his math tutor, accountant, ship designer. He used his knowledge of astronomy/astrology to provide navigational expertise.

He made only one expedition to the Americas, around 1585-86, Roanoke Island (North Carolina). He learned the Algonquin language, and wrote about this voyage in 1588. He is sometimes credited with the introduction of thepotato to Great Britain and Ireland.

During the voyage, Harriot was asked by Raleigh to find the most efficient way to stack cannon balls on the deck of the ship. His ensuing theory about the close-packing of spheres, and his correspondence with Kepler, led to Kepler's Conjecture that the most efficient packing was the usual one. (Solved in 1998.)

Harriot was the first person to make a drawing of the Moon through a telescope, on July 26, 1609.

Made a careful study of Vieta's work, and was converted to the use of symbolism.

Albert Girard (1595–1632)

He was the first to use the Trigonometry abbreviations 'sin', 'cos' and 'tan' in a treatise.
1629 *A New Discovery in Mathematics* – first explicit statement of the Fundamental Theorem of Algebra.

Modern notation for $\sqrt[3]{a}$ but wrote $(\frac{3}{2})a$ and $a(\frac{3}{2})$ for $a^{3/2}$ and $ax^{3/2}$

Elementary symmetric functions of n variables "factions"

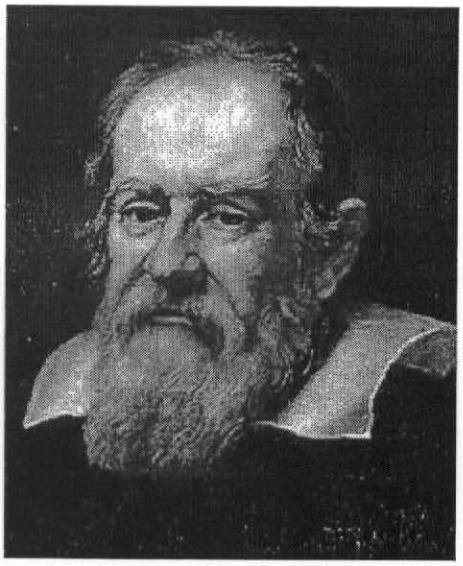
$x_1 + \cdots + x_n$ = first faction

$x_1x_2 + \cdots + x_ix_j + \cdots + x_{n-1}x_n$ = second faction

} # of terms in k^{th} faction is $\binom{n}{k}$

Galileo Galilei (1564-1642)

Galileo di Vincenzo Bonaiuti de' Galilei
In 1589, he was appointed to the chair of mathematics in Pisa



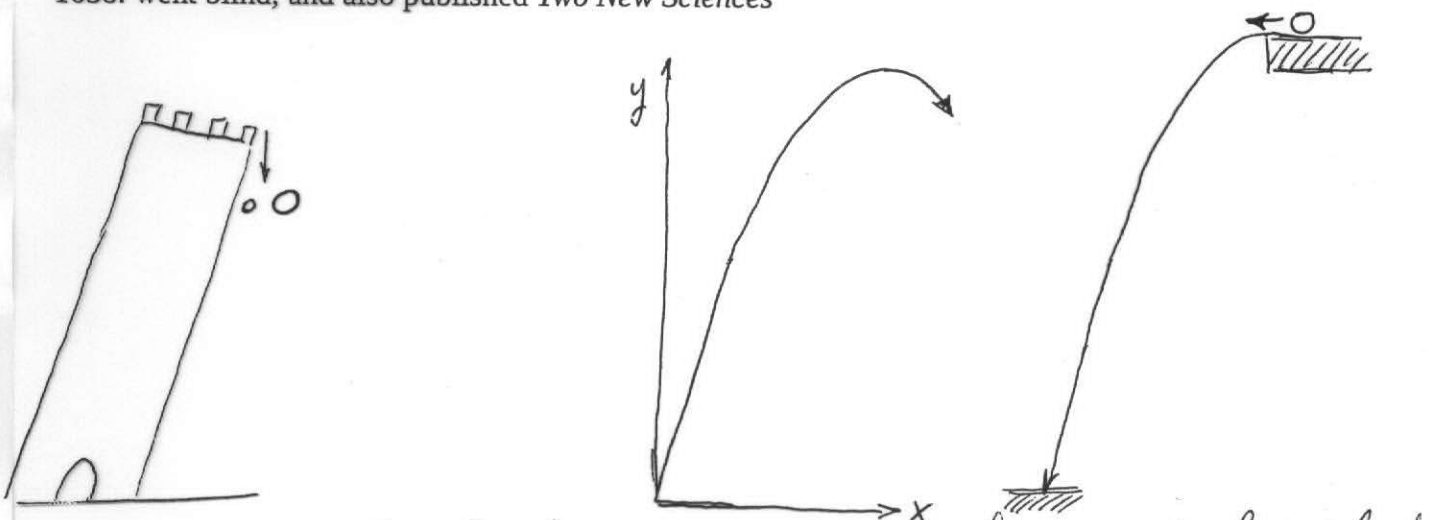
1604: falling body has constant acceleration. Tower of Pisa!
Worked out mathematical consequence: $x=Ct^2$

1608: Worked out projectile motion.

In 1610 Galileo published an account of his telescopic observations of the moons of Jupiter, using this observation to argue in favor of the sun-centered, Copernican theory of the universe. He then went to Rome to demonstrate the telescope to the Pope and others.

1632 (age 68): put under house arrest, tried and convicted of heresy for promoting Sun-centered viewpoint. From 1634-1642 he was placed under house arrest in the countryside near Florence.

1638: went blind, and also published *Two New Sciences*



Mathematical Modelling of Nature

Consider most important ideas, abstract them
Work out mathematical consequences
Compare to experiment

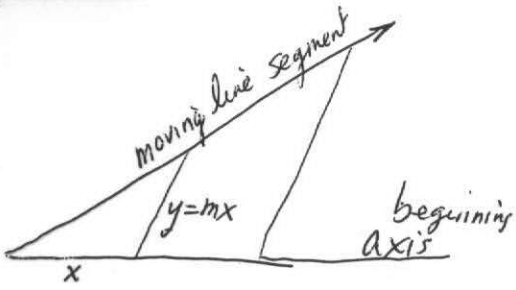
Compounded from equal-motion horizontal motion (fundamental law of inertia – no force) and naturally accelerated downward motion

Pierre de Fermat (1601-1655)

Went to University of Toulouse (SW France), then Bordeaux.
1630-31: studied civil law at the University of Orléans.
Given title of councillor at the High Court in Toulouse,

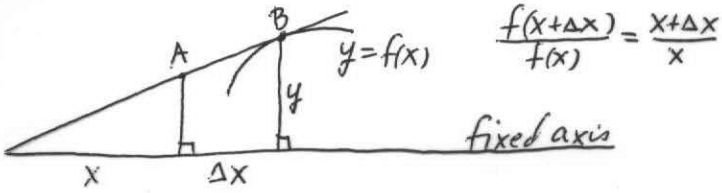
Ad Locos Planos et Solidos Isagoge (Introduction to plane and solid loci) was circulated in manuscript form in 1636, predating the publication of Descartes' famous *La géométrie*. This manuscript was published posthumously in 1679.

Fermat used only the x-axis and considered a perpendicular moving along the line, whose length y changed according to a formula. He showed that the equations $y=mx$ and $y=x^2+bx+c$ gave lines and parabolas.



1638

Fermat developed a method for determining maxima, minima, and tangents to various curves that was equivalent to differentiation. (Chapter II)



In number theory

Most famous for Fermat's Last Theorem:t $x+y=z$ has no solutions if $n>2$.
He solved this for $n=3$ and $n=4$.
Fermat also studied Pell's equation ($y^2=dx^2+b$), Fermat primes (1+power of 2), ...
While studying perfect numbers he discovered Fermat's Little Theorem: $n!=-1 \pmod n$
He also invented the proof technique of infinite descent, an early form of induction.

René Descartes (1596-1650)

His father wanted him to be a lawyer, so after attending Jesuit school he got a law degree in Poitiers in 1616. Then he "dropped out" until 1632, when he settled in the Netherlands. He became famous and lived comfortably.

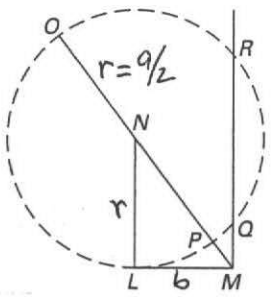
Descartes created analytic geometry. (Cartesian coordinates)

1637: Discourse on the Method for Rightly Directing one's Reason and Searching for Truth in the Sciences.

Geometric description of a curve is primary, the algebraic equation describing it is secondary.

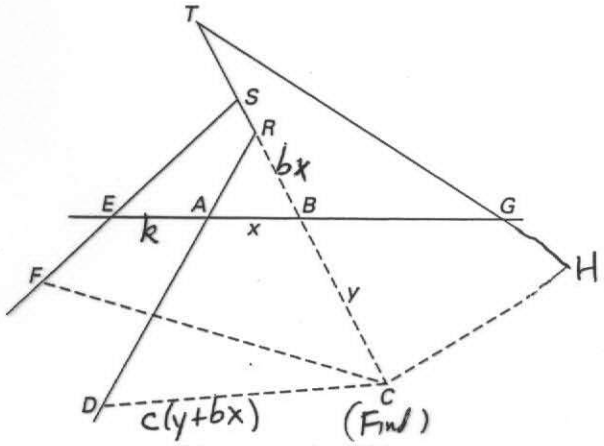
Solve $z^2 = az + b^2$

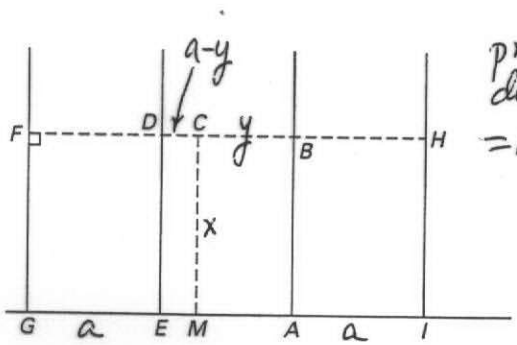
$z = \frac{1}{2}a + \sqrt{\frac{1}{4}a^2 + b^2}$.



Descartes { unknowns ⟷ x, y, z
Notation { constants ⟷ a, b, c }

ax^3 used instead of $a^{(3)}$





product of horizontal distances to 3 lines = a(dist to other 2)

$(2a - y)(a - y)(y + a) = ayx$, or $y^3 - 2ay^2 - a^2y + 2a^3 = axy$.

Apollonius' Problem (Known to A.)

Given 4 lines, find all points C

Distance to each line is linear $(ax+by+c)$

$\frac{d_1 d_2}{d_3 d_4}$ = Constant gives a conic