



Rhind papyrus

Rhind papyrus Problem 28. A quantity together with its two-thirds has one third its sum taken away to yield 10. What is the quantity?

Says " $x + (2/3)x - (x + (2/3)x)/3 = 10$. What is x ?" Restated, divide 10 by $1 + 2/3 - (1 + 2/3)/3$, or 10 by $1 + 1/3 + 1/6 + 1/18$ (see 2/n table).

- 1 $1 + 1/3 + 1/6 + 1/18$
- 2 $3 + 1/9$ (see 2/n table)
- 4 $6 + 1/6 + 1/18$ (see 2/n table)

Now if we double once more we get a number, $12 + 1/3 + 1/9$, greater than 10, so the Egyptians asked the question, "What can be added to $6 + 1/6 + 1/18$ to get 10?" They deduced the answer to be $3 + 1/2 + 1/6 + 1/9$. Then they asked, "What must be multiplied by $1 + 1/3 + 1/6 + 1/18$ to get $3 + 1/2 + 1/6 + 1/9$?" This was deduced to be $2 + 1/4 + 1/28$. So the solution is

$$4 + 2 + 1/4 + 1/28 = 6 + 1/4 + 1/28.$$

Rhind papyrus Problem 79. There are seven houses; in each house there are seven cats; each cat kills seven mice; each mouse has eaten seven grains of barley; each grain would have produced seven hekat. What is the sum of all the enumerated things.

houses	7		
cats	49		
mice	343	1	2801
heads of barley	2401	2	5602
hekats of barley	16807	4	11204
total	19607	total	19607

The first two columns leads to the sum (in the bottom row) of the five terms of the geometric sequence with ratio 7 beginning with 7:

$7 + 7^2 + 7^3 + 7^4 + 7^5$. While the second to columns is the usual method for multiplying 7×2801 . Finally, it is observed that the former sum equals the latter product. To an archeologist the table above and the relationship between the two columns may be meaningless, and several have said this. However, to an arithmetician, the relationship between the two columns is clear since (we know) the formula for the geometric series of the first n terms of a geometric series

$$\{1, r, r^2, \dots, r^n\}$$

$$x(1 + 1/3 + 2/9) = 10 = y$$

$$\text{If } x=4, y=6 \frac{2}{9}$$

$$\text{So need } x(1 + 1/3 + 2/9) = 3 \frac{7}{9}$$

$$\text{If } x=2, y=3 \frac{1}{9}$$

$$\text{So need } x(1 + 1/3 + 2/9) = 6/9 = 2/3$$

$$\text{If } x=1/4, y=1/4 + 1/12 + 1/18$$

$$\text{If } x=1/28, y=1/128 + 1/84 + 1/126$$

Seven times

$$1 + r + r^2 + r^3 + r^4 = \frac{r^5 - 1}{r - 1}$$

$$7^5 = 16807$$

$$\frac{7^5 - 1}{6} = \frac{16806}{6} = 2801$$

$$7 \times 2801 = 19607$$