



Rhind papyrus

Rhind papyrus Problem 28. A quantity together with its two-thirds has one third its sum taken away to yield 10. What is the quantity?

Says " $x + (2/3)x - (x + (2/3)x)/3 = 10$ . What is  $x$ ?" Restated, divide 10 by  $1 + 2/3 - (1 + 2/3)/3$ , or 10 by  $1 + 1/3 + 1/6 + 1/18$  (see 2/n table).

- 1  $1 + 1/3 + 1/6 + 1/18$
- 2  $3 + 1/9$  (see 2/n table)
- 4  $6 + 1/6 + 1/18$  (see 2/n table)

Now if we double once more we get a number,  $12 + 1/3 + 1/9$ , greater than 10, so the Egyptians asked the question, "What can be added to  $6 + 1/6 + 1/18$  to get 10?" They deduced the answer to be  $3 + 1/2 + 1/6 + 1/9$ . Then they asked, "What must be multiplied by  $1 + 1/3 + 1/6 + 1/18$  to get  $3 + 1/2 + 1/6 + 1/9$ ?" This was deduced to be  $2 + 1/4 + 1/28$ . So the solution is

$$4 + 2 + 1/4 + 1/28 = 6 + 1/4 + 1/28.$$

Rhind papyrus Problem 79. There are seven houses; in each house there are seven cats; each cat kills seven mice; each mouse has eaten seven grains of barley; each grain would have produced seven hekat. What is the sum of all the enumerated things.

|                  |       |       |       |
|------------------|-------|-------|-------|
| houses           | 7     |       |       |
| cats             | 49    |       |       |
| mice             | 343   | 1     | 2801  |
| heads of barley  | 2401  | 2     | 5602  |
| hekats of barley | 16807 | 4     | 11204 |
| total            | 19607 | total | 19607 |

The first two columns leads to the sum (in the bottom row) of the five terms of the geometric sequence with ratio 7 beginning with 7:

$7 + 7^2 + 7^3 + 7^4 + 7^5$ . While the second to columns is the usual method for multiplying  $7 \times 2801$ . Finally, it is observed that the former sum equals the latter product. To an archeologist the table above and the relationship between the two columns may be meaningless, and several have said this. However, to an arithmetician, the relationship between the two columns is clear since (we know) the formula for the geometric series of the first  $n$  terms of a geometric series

$$\{1, r, r^2, \dots, r^n\}$$

$$x(1 + 1/3 + 2/9) = 10 = y$$

$$\text{If } x=4, y=6 \frac{2}{9}$$

$$\text{So need } x(1 + 1/3 + 2/9) = 3 \frac{7}{9}$$

$$\text{If } x=2, y=3 \frac{1}{9}$$

$$\text{So need } x(1 + 1/3 + 2/9) = 6/9 = 2/3$$

$$\text{If } x=1/4, y=1/4 + 1/12 + 1/18$$

$$\text{If } x=1/28, y=1/128 + 1/84 + 1/126$$

Seven times

$$1 + r + r^2 + r^3 + r^4 = \frac{r^5 - 1}{r - 1}$$

$$7^5 = 16807$$

$$\frac{7^5 - 1}{6} = \frac{16806}{6} = 2801$$

$$7 \times 2801 = 19607$$

Rhind papyrus Problem 72. 100 loaves of pesu 10 are to be exchanged for a certain number of loaves of pesu 45. What is the number?

The pesu number of a bread determined its strength in inverse order; in particular, pesu 10 is stronger/better than pesu 45 with the number determining some kind of percent of something undesired. Thus the problem reads in our modern sense as "What is  $(45/10)100$ ?" Obviously its 450. Here's the Egyptian solution:

First find the excess of 45 over 10. You get 35. Divide 35 by 10. You get  $3 + 1/2$ . Now multiply  $3 + 1/2$ . You get 350. Add 100 to 350, and get 450.

The above may seem odd, but it is algebraically correct. Suppose we have  $x$  loaves of pesu  $p$  and  $y$  loaves of  $x$  loaves of pesu  $p$  is to be exchanged for pesu  $q$ .

Problem 75 How many loaves of pesu 30 can be made from same amount of flour as 155 loaves of pesu 20?

$$x = \frac{155 \times 30}{20} = 232 \frac{1}{2}$$

$$x = 100 \times \frac{45}{10} = 100 + 300 \times \frac{35}{10} = 450$$

### SIMPLE EQUATIONS

Rhind papyrus Problem 24. A quantity (any) plus one-seventh of it becomes 19. What is the quantity?

Says " $x + x(1/7) = 19$ . What is  $x$ ?" Restated, divide 19 by  $1 + 1/7$ . Of course, we would write

$$1 + 1/7 = 8/7 \text{ and } 19/(8/7) = 133/8 = 16 + 5/8$$

|    |  |
|----|--|
| 1  | $1 + 1/7$                                    |
| 2  | $2 + 2/7 = 2 + 1/4 + 1/28$ (see 2/n table)   |
| 4  | $4 + 1/2 + 1/14$                             |
| 8  | $9 + 1/7$                                    |
| 16 | $18 + 2/7 = 18 + 1/4 + 1/28$ (see 2/n table) |

Now if we double once more we get a number,  $36 + 1/2 + 1/14$ , greater than the numerator, so the Egyptians asked the question, "What can be added to  $18 + 1/4 + 1/28$  to get 19?" They deduced the answer to be  $1/2 + 1/7 + 1/14$ . Then they asked, "What must be multiplied by  $1 + 1/7$  to get  $1/2 + 1/7 + 1/14$ ?" This was deduced to be  $1/2 + 1/8$ . So the solution is  $16 + 1/2 + 1/8 = 16 + 5/8$ .

Rhind papyrus Problem 64. Divide 10 hekats of barley among 10 men so that the common difference is  $1/8$  of a hekat of barley.

Their solution is as follows: Average the value  $10/10 = 1$ . The total number of differences is then  $10 - 1 = 9$ . Find half the common difference,  $(1/2)(1/8) = 1/16$ . Multiply 9 by  $1/16$ :  $1/16 + (8 \text{ by } 16) = 1/2 + 1/16$ . Add this to the average value to get the largest share  $1 + 1/2 + 1/16$ . Subtract the common difference,  $1/8$ , nine times [ $9 \times (1/8) = 1 + 1/8$ ] to get the lowest share

$$(1 + 1/2 + 1/16) - (1 + 1/8) = 1/2 + 1/8 + 1/16. \text{ Thus, the shares are } 1/2 + 1/8 + 1/16,$$

$$1/2 + 1/4 + 1/16, 1/2 + 1/4 + 1/8 + 1/16, 1 + 1/16, 1 + 1/8 + 1/16, 1 + 1/4 + 1/16,$$

$$1 + 1/4 + 1/8 + 1/16, 1 + 1/2 + 1/16. \text{ The total is 10 hekats of barley.}$$

Given the last statement in the solution, we can conclude the Egyptians knew that the sum  $S$  of the first  $n$  terms of an arithmetic sequence starting with  $s$  and common difference  $d$ ,

$$\{s, s+d, s+2d, \dots, s+d+2d+\dots+(n-1)d\},$$

$$\text{is } sn + (1/2)dn(n-1), \text{ or as applied above } S/n = s + d(n-1)/2.$$

$$x(1 + 1/7) = 19$$

$$\text{Try } x = 1, 2, 4, 8, 16$$

$$x = 16 \text{ yields } 18 \frac{1}{4} + \frac{1}{128}$$

$$y(1 + 1/7) = \frac{1}{2} + \frac{1}{7} + \frac{1}{128}$$

$$y = \frac{1}{2} + \frac{1}{8}$$

$$\boxed{x + y = 16 + \frac{1}{2} + \frac{1}{8}}$$