

MATH 454 PRACTICE SECOND MID-TERM

Question 1. Find the coefficient of x^9 in the expansion of $(1 + 5x)^{-1/5} x^6$.

Question 2. Solve the recurrence

$$a_n = 8a_{n-1} - 20a_{n-2} + 16a_{n-3}, \quad n \geq 3$$

when $a_0 = 1$, $a_1 = 6$ and $a_2 = 24$. (Hint: 2 is a characteristic root.)

Question 3. Find the number of codewords of length k from the alphabet $\{a, b, c, d, e\}$ if a and b occur an even number of times and c occurs an odd number of times.

Question 4. Use the ordinary generating function to solve the recurrence

$$(1) \quad a_{k+1} = 5a_k + 7^k, \quad k \geq 1$$

$$(2) \quad a_1 = 6.$$

Question 5. A subset $S \subseteq \{1, 2, \dots, n\}$ is said to be *unfriendly* iff S does not contain any pairs of consecutive numbers. (For example, $\{1, 3, 6\}$ is unfriendly and $\{1, 3, 5, 6, 8\}$ is not unfriendly.) Find a recurrence for the number a_n of unfriendly subsets of $\{1, 2, \dots, n\}$.