Calculators will not be used on the exam; you will be expected to leave answers in unsimplified form. To enable you to check your work here, though, I have given you numerical answers for some of the problems.

1. Which statements below are always true? Never true? Sometimes true and sometimes false? For each "sometimes" statement, give an example in which it is true and one in which it is false.
(a) $P(E \cup F) < P(E) + P(F)$; (b) $P(E \cup F) = P(E) + P(F)$; (c) $P(E \cup F) > P(E) + P(F)$;
(d) $P(E \cap F) < P(E)P(F)$; (e) $P(E \cap F) = P(E)P(F)$; (f) $P(E \cap F) > P(E)P(F)$.

2. Five young women and three young men are close friends. One night, each of the women calls one of the men, whom she chooses at random. Find the probability that exactly k men are called, for $k = 1$, 2, and 3.

3. A die is rolled twice. Let E be the event that the sum of the two numbers obtained is seven; F the event that the first number obtained is even, and G the event that the second number obtained is odd. Are E and F independent? E and G? F and G? E, F, and G?

4. Suppose we have five urns numbered 1 through 5 and that urn k contains k red and $5 - k$ black balls. An urn is selected at random and a ball drawn from it at random; the ball is red. What is the probability that urn 3 was selected?

5. Each morning, Fred makes a random choice of one of three routes to take to work. After n trips, ($n > 0$), what is the probability that he has traveled each route at least once? Hint: use inclusion-exclusion to calculate the probability of the complementary event.

6. A die is rolled n times. What is the probability that at least one of the numbers 1 through 6 does not appear on any of the rolls?

7. Suppose that E and F are two mutually exclusive events, each of which has nonzero probability. Under what circumstances are F and G independent?

8. Suppose that n men and n women are seated at a round table, with seats assigned at random. (a) What is the probability that at least one pair of men will be in adjacent seats? Check your answer by showing that for large n this probability is approximately $1-\sqrt{n\pi}/2^{2n-1}$. (b) One of the men is married to one of the women. What is the probability that this couple is seated in adjacent seats? Directly across the table from one another?

9. What is the probability that a poker hand will contain a card from every suit? What is the probability of this if we know that the hand contains at least two hearts?

10. An urn contains six red and three white balls. Alice and Bob draw balls in turn, with Alice drawing first. The game ends when two white balls have been drawn; the player who draws the *second* white ball is the winner. What is the probability that Alice wins: (a) If the draws are without replacement? (b) If the draws are with replacement? (Hint: condition on the first draw.)

11. (a) Let X be the number of aces in a poker hand. Find the probability mass function of X.

(b) Five cards are obtained by drawing one card at random from each of five decks; let Y be the number of aces obtained. Find the probability mass function of Y.

12. An urn contains two red and two white balls; balls are drawn out at random until two red balls are obtained. Let X be the number of draws required. Find the probability mass function, cumulative distribution function, expectation, and variance of X.

Some answers and hints (not checked carefully; be suspicious):
1. All but one are "sometimes". 2. 0.01234, 0.37037, 0.61728. 3. Y, Y, Y, N. 4. 0.2.
5. $(3^{n-1} - 2^n + 1)/3^{n-1}$. 6. If $n = 20$: 0.15201. 8.(b) If $n = 20$: 0.0513, 0.0256.
9. 0.2637; 0.1796. 10. 0.47619; 0.48000. 11. $P(X = 2) = 0.03993$, $P(Y = 2) = 0.04654$.
12. $E[X] = 3.3333$; $\mathrm{Var}(X) = 0.5555$.