# Some Random Variables

Math 477:02 — Spring 2004

## Discrete Random Variables

| Random Variable | Situation Modeled | Parms | E($X$) | Var($X$) | Probability Mass Fcn |
|---|---|---|---|---|---|
| Bernoulli | Flip a (not nec fair) coin.$p$ ist the probability of heads ("success"). $q = 1 - p$. | $p$ | $p$ | $pq$ | $P\{X = i\} = \begin{cases} q & \text{if } i = 0 \\ p & \text{if } i = 1 \end{cases}$ |
| Binomial | Count successes in a fixed number $n$ of independent Bernoulli trials, each with probability $p$ of success. | $p$, $n$ | $np$ | $npq$ | $P\{X = i\} = \binom{n}{i} p^i q^{n-i}$ |
| Geometric | The number of repetitions until the first success in independent Bernoulli trials with parameter $p$. | $p$ | $\frac{1}{p}$ | $\frac{q}{p^2}$ | $P\{X = i\} = p\, q^{i-1}$ |
| Negative Binomial | The number of independent Bernoulli trials with parameter $p$ until there are $r$ successes. | $p$, $r$ | $\frac{r}{p}$ | $\frac{rq}{p^2}$ | $P\{X = i\} = \binom{i-1}{r-1} p^r\, q^{i-r}$ |
| Hypergeometric | The number of white balls in a set of $n$ balls selected from $N$ balls of which $m$ are white. $p = \frac{m}{N}$ = probability of drawing 1 white ball. | $n$, $N$, $m$ | $\frac{nm}{N}$ $= np$ | $\frac{N-n}{N-1}\, \frac{nm}{N} \left(1 - \frac{m}{N}\right)$ $= \frac{N-n}{N-1}\, n\, p\, q$ | $P\{X = i\} = \frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}}$ |
| Poisson | Approximate Binomial where $n >> 1$, $\lambda = np$ moderate.<br><br>*Poisson process:* (i) The probability that precisely 1 instance of the outcome being looked at occurs in a given interval of length $h$ is $\lambda h + o(h)$;<br>(ii) The probability that 2 or more instances occur in a given interval of length $h$ is $o(h)$; and<br>(iii) For any set $\{S_i\}$ of nonoverlapping intervals, if $E_i$ is the event that $j_i$ instances occur in $S_i$, then $\{E_i\}$ are independent events.<br>Then the number of instances of the outcome being looked at occuring in an interval of length $t$ is a Poisson RV with parameter $\lambda t$. | $\lambda$ | $\lambda$ | $\lambda$ | $P\{X = i\} = e^{-\lambda}\, \frac{\lambda^i}{i!}$ |

**Continuous Random Variables**

| Rand. Var. | Situation Modeled | Par. | E($X$) | Var($X$) | Prob density function | Cumulative Distribution Function |
|---|---|---|---|---|---|---|
| Uniform | Constant probability density on an interval $[a, b]$. | | $\frac{a+b}{2}$ | $\frac{(b-a)^2}{12}$ | $f(x) = \begin{cases} 0 & \text{if } x \le a \\ \frac{1}{b-a} & \text{if } a < x \le b \\ 0 & \text{if } b < x \end{cases}$ | $F(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{1}{b-a}(x-a) & \text{if } a \le x \le b \\ 1 & \text{if } b < x \end{cases}$ |
| Normal | Excellent approximation to a Binomial RV with the same mean $\mu$ and the same variance $\sigma^2$. Models many situations. Tables used for values of its c.d.f. $\Phi$. | $\mu$, $\sigma$ | $\mu$ | $\sigma$ | $f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ for $-\infty < x < \infty$ | $\Phi\left(\frac{x-\mu}{\sigma}\right)$ from tables<br>$\Phi(z) = 1 - \Phi(-z)$ if $z < 0$ |
| Exponential | The amount of time until the first occurence of an event in a Poisson process with density parameter $\lambda$ The exponential RV is the only *memoryless* RV, that is, the only RV with $\mathrm{P}\{X > r+s \mid X > s\} = \mathrm{P}\{X > r\}$ | $\lambda$ | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ | $f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ \lambda\, e^{-\lambda x} & \text{if } x > 0 \end{cases}$ | $F(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 1 - e^{-\lambda x} & \text{if } x > 0 \end{cases}$ |
| Gamma | The amount of time one has to wait for the $t^{th}$ occurrence of a Poisson process outcome with density parameter $\lambda$ in the case that $\alpha = n$ is a positive integer. $\Gamma(n) = (n-1)!$ for $n$ a positive integer | $\alpha$, $\lambda$ | $\frac{\alpha}{\lambda}$ | $\frac{\alpha}{\lambda^2}$ | $f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ \frac{\lambda\, e^{-\lambda x}\,(\lambda x)^{\alpha-1}}{\Gamma(\alpha)} & \text{if } x > 0 \end{cases}$ | $F(x) = \begin{cases} 0 & \text{if } x \le 0 \\ \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^x u^{\alpha-1}\, e^{-u\lambda}\, du & \text{if } x > 0 \end{cases}$ |