Math 477:02 – Spring 2004 Answers to Homework for 1/29/04

You need not do the arithmetic done here. You may leave your answers in combinatorial notation form.

Chapter 1, P15: There are $\binom{12}{5}$ ways to select 5 men from a group of 12, and $\binom{10}{5}$ ways to select 5 women from a group of 10. Once you have obtained the two groups of 5, line up the men and then assign women to them in order to get 5! possible ordered pairs, so by the multiplication principle, there are

$$\binom{12}{5} \times \binom{10}{5} \times 5! = \frac{12 \times 11 \times 10^2 \times 9^2 \times 8^2 \times 7 \times 6 \times 120}{(120)^2} = 23,950,080$$

possible results of this process.

Chapter 1, P20: (a) There are $\binom{6}{5}$ ways of issuing invitations that do not include either of the feuding parties; and $\binom{6}{4}$ ways to invite feuding party A but not B; and $\binom{6}{4}$ ways to invite feuding party B but not A.Since these are mutually exclusive, there are

$$\binom{6}{5} + \binom{6}{4} + \binom{6}{4} = \frac{6}{1} + 2 \times \frac{6 \times 5}{2} = 6 + 30 = 36$$

ways of inviting the 5 friends.

There is another way to do this. There are $\binom{8}{5}$ ways of inviting 5 friends to the party, and $\binom{6}{3}$ of these include both A and B. Hence there are $\frac{8\times7\times6\times5\times4}{120} - \frac{6\times5\times4}{6} = 56 - 20 = 36$ ways of inviting 5 friends but not including both A and B.

(b): There are $\binom{6}{3}$ ways of inviting A and B and choosing 3 friends from the remaining 6 to invite, and there are 6 ways of choosing 5 friends distinct from A and B to invite. Thus there are

$$\frac{6 \times 5 \times 4}{6} + 6 = 26$$

ways of inviting 5 friends either including both A and B or excluding them both.

Chapter 1, P28: Each new teacher can be assigned to 4 different schools, so there are $4^8 = 65,536$ ways to assign the teachers. If each school must receive 2 teachers, there are $\binom{8}{2}$ ways to select 2 for the first school; then $\binom{6}{2}$ ways to select 2 from those left to go to the second school, $\binom{4}{2}$ ways to select for the third school, and 1 way for the 4^{th} school. By the multiplication principle, there are

$$\binom{8}{2} \times \binom{6}{2} \times \binom{4}{2} = 28 \times 15 \times 6 = 2520$$

ways to place 2 at each school.

Chapter 1, TE14: (a) The number of ways of selecting j committee members out of the n people is $\binom{n}{j}$. After that one can select the subcommittee $\binom{j}{i}$ ways. By the multiplication principle, there are $\binom{n}{j} \times \binom{j}{i}$ ways to get the committee and subcommittee.

There are $\binom{n}{i}$ ways of selecting the subcommittee from the n people, and then $\binom{n-i}{j-i}$ ways of selecting the remaining $j-i$ members of the committee. By the multiplication principle, there are $\binom{n}{i} \times \binom{n-i}{j-i}$ ways to get the committee and subcommittee. Thus

$$\binom{n}{j} \times \binom{j}{i} = \binom{n}{i} \times \binom{n-i}{j-i}.$$

(b): Sum both sides of the equations in (a) letting j go from i to n, and then setting $k = j - i$.

$$\sum_{j=i}^{n} \binom{n}{j} \times \binom{j}{i} = \sum_{j=i}^{n} \binom{n}{i} \times \binom{n-i}{j-i} = \binom{n}{i} \times \sum_{j=i}^{n} \binom{n-i}{j-i}$$

$$= \binom{n}{i} \times \sum_{k=0}^{n-i} \binom{n-i}{k} = \binom{n}{i} \times 2^{n-i}$$

since $(1+1)^{n-i} = \sum_{k=0}^{n-i} \binom{n-i}{k} \times 1^k \times 1^{n-i-k} = \sum_{k=0}^{n-i} \binom{n-i}{k}$.