Math 477:02 – Spring 2004 Answers to Homework for 2/5/04

You need not do the arithmetic done here. You may leave your answers in combinatorial notation form.

Chapter 2, P8: Suppose A and B are mutually exclusive events for which $P(A) = .3$ and $P(B) = .5$. What is the probability that

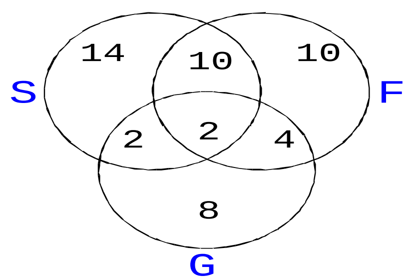
(a) either A or B occurs? $P(A \cup B) = .3 + .5 = .8$ **by countable additivity (axiom 3).**

(b) A occurs but B does not? **If A occurs, B cannot occur, so the probability that A occurs but B does not is** $P(A) = .3$.

(c) both A and B occur. **They are mutually exclusive events, so** $P(AB) = 0$.

Chapter 2, P12: A school has classes in Spanish, German, and French. There are 100 students in the school, 28 in Spanish, 26 in French, 16 in German, 12 in both Spanish and French, 4 in both Spanish and German, 6 in both French and German, and 2 in all 3.

We initially find out how many students are taking which languages via a Venn diagram. The numbers in the shapes below indicate how many are in each set. This was obtained by putting in the 2 for students in all 3, then computing the remaining students in each pair of languages, and then computing the remaining students taking only one language.



We check our work by using inclusion/exclusion. Let S (Spanish), G (German), and F (French) be the event consisting of students taking the indicated language. Then $\#(\mathrm{S} \cup \mathrm{G} \cup \mathrm{F}) = \#(\mathrm{S}) + \#(\mathrm{G}) + \#(\mathrm{F}) - (\#(\mathrm{SG}) + \#(\mathrm{SF}) + \#(\mathrm{GF})) + \#(\mathrm{SGF}) = 28 + 16 + 26 - (4 + 12 + 6) + 2 = 50$ **and the sum of the numbers in the disjoint areas above is** $14 + 10 + 8 + 10 + 2 + 2 + 4 = 50$.

Now we are ready to answer the questions asked.

(a) If a student is chosen randomly, what is the probability that he or she is not in any of these classes? 50 **out of** 100 **students are in language classes, so** 50 **are not. Thus** P **(not in a language class)** $=$ 0.5.

(b) If a student is chosen randomly, what is the probability that he or she is precisely in one of these classes? $\#$ **(exactly 1 language)** $= 14 + 10 + 8 = 32$ **from the picture, or, by inclusion/exclusion**

$$\begin{aligned}
\#\,(\textbf{exactly 1 language}) &= 50 - \#(\mathrm{SG} \cup \mathrm{SF} \cup \mathrm{GF}) \\
&= 50 - \left(\begin{array}{c} \#(\mathrm{SG}) + \#(\mathrm{SF}) + \#(\mathrm{GF}) \\ -(\#(\mathrm{SGSF}) + \#(\mathrm{SGGF}) + \#(\mathrm{SFGF})) + \#(\mathrm{SGSFGF}) \end{array} \right). \\
&= 50 - (4 + 12 + 6 - (2 + 2 + 2) + 2) = 32.
\end{aligned}$$

Thus P **(exactly 1 language)** $= \frac{32}{100}$.

(c) If 2 students are chosen randomly, what is the probability that at least one of them is taking a language class? **Select one of the students. The probability that this student is taking a language class is $\frac{1}{2}$. Now consider the disjoint event that the first student is not taking a language class but the second one is. This occurs with probability $\frac{1}{2} \times \frac{50}{99} = \frac{25}{99}$. Thus**

$$P\,(\textbf{at least one taking language}) = \frac{1}{2} + \frac{25}{99} = \frac{149}{198} = 0.75253.$$

There is another way to get this. Look at all ordered pairs of distinct students. There are $P_2^{100} = 100 \times 99 = 9900$ of them. Let E_1 be the event that the first student in the pair is taking language, and E_2 the event that the second student in the pair is taking language. Then $\#(E_1E_2) = \#(E_1) + \#(E_2) - \#(E_1E_2) = 50 \times 99 + 99 \times 50 - 50 \times 49 = 7450$ so $P\,(\textbf{at least one taking language}) = \frac{7450}{9900} = \frac{149}{198}$

Chapter 2, P28: An urn contains 5 R(ed), 6 B(lue), and 8 G(reen) balls. A set of 3 balls is randomly selected. Initially it is done with replacement, then it is also done without replacement. What is the probability that:

(a) all of the balls will be of the same color.

i. $P\,(\textbf{all same color without replacement}) = P\,(3R) + P\,(3B) + P\,(3G) = \left(\frac{5\cdot4\cdot3}{19\cdot18\cdot17}\right) + \left(\frac{6\cdot5\cdot4}{19\cdot18\cdot17}\right) + \left(\frac{8\cdot7\cdot6}{19\cdot18\cdot17}\right) = \frac{86}{969} = 8.8751 \times 10^{-2}$

ii. $P\,(\textbf{all same color with replacement}) = P\,(3R) + P\,(3B) + P\,(3G) = \left(\frac{5}{19}\right)^3 + \left(\frac{6}{19}\right)^3 + \left(\frac{8}{19}\right)^3 = \frac{853}{6859} = 0.124\,36$

(b) no two balls will have the same color. First we assume that the balls are picked in order first R and then B and then G. Changing the order changes the order of the factors in the numerator thus giving the same probability for any of the 6 distinct permutations.

i. $P\,(\textbf{all different colors without replacement}) = 3!\left(\frac{5\cdot6\cdot8}{19\cdot18\cdot17}\right) = \frac{80}{323} = 0.24768$

ii. $P\,(\textbf{all different colors with replacement}) = 3!\left(\frac{5\cdot6\cdot8}{19^3}\right) = \frac{1440}{6859} = 0.20994$

Chapter 2, P39: There are 5 hotels in a certain town. If 3 people check into hotels in a day, what is the probability they each check into a different hotel. What assumptions are you making.

Assume that any one check in is as likely as any other check in. That is, assume checking in is completely random with no person having any reason to prefer one hotel over another, and all hotels have available rooms.

The first person can check into any of 5 hotels, then the second can check into any of 4 hotels, and then the third can check into any of 3 hotels if no two check into the same hotel. There are 5^3 ways 3 people can check into 5 hotels. Thus $P\,(\textbf{different hotels}) = \frac{5\cdot4\cdot3}{125} = \frac{12}{25}$

Chapter 2, TE9: Suppose that an experiment is performed n times. For any event E of the sample space, let $n(E)$ denote the number of times that event E occurs, and define $f(E) = \frac{n(E)}{n}$. Show that $f(E)$ satisfies the axioms of probability.

Axiom 1: **Since the number of times any event can occur in n repetitions of the experiment is a number between 0 and 1, $0 \leq \frac{n(E)}{n} \leq 1$.**

Axiom 2: **The sample space S of an experiment must contain all possible outcomes of the experiment. Hence $n(S) = n$ and $\frac{n(S)}{n} = \frac{n}{n} = 1$.**

Axiom 3: **Let $\{E_i : i \in \omega\}$ be a countable family of mutually exclusive events. Since any outcome occurs in at most one of the E_i, $n\left(\bigcup_{i=0}^{\infty} E_i\right) = \sum_{i=0}^{\infty} n(E_i)$, so $\frac{n\left(\bigcup_{i=0}^{\infty} E_i\right)}{n} = \frac{\sum_{i=0}^{\infty} n(E_i)}{n} = \sum_{i=0}^{\infty} \frac{n(E_i)}{n}$.**