Math 477:02 March 8, 2004 Name: ____________________________

Instructions: No books, notes, calculators, or other aids are permitted. Leave answers as binomial coefficients or fractions rather than decimals, and use complete sentences of the form “The probability that <the event in the question> is <your answer as a fraction>.” to give your answers to the questions asked.

(a) What is the probability that a bridge hand of 13 cards from a 52 card deck is void in at least one suit.

Note: The answer is NOT \( \binom{4}{1} \binom{39}{13} / \binom{52}{13} \)

At least should trigger the thought inclusion/exclusion. Let \(E_i\) be the event that the hand is void in suit \(i\) for \(i \in \{1, 2, 3, 4\}\). Then \(E = \bigcup_{i=1}^{4} E_i\) and \(P(E_i) = \binom{39}{13} / \binom{52}{13}\). By inclusion/exclusion

\[
P(E) = \binom{4}{1} \binom{39}{13} - \binom{4}{2} \binom{26}{13} + \binom{4}{3} \binom{13}{13}
\]

That is: The probability that there is at least one void is \(\binom{4}{1} \binom{39}{13} - \binom{4}{2} \binom{26}{13} + \binom{4}{3} \binom{13}{13} / \binom{52}{13}\).

(b) A bridge hand is dealt, so each of 4 players has 13 cards from the 52 card deck. You have 5 spades in your hand. What is the probability that at least one of the other three hands is void in spades. (For those who know bridge, no bidding has as yet occurred.)

The previous question was taken directly from a problem in the book. This one still uses those words “at least” so again you expect to need inclusion exclusion. There are 39 cards not included in your hand, including 8 spades. Let \(E_i\) be the event that hand \(i\) is void in spades for \(i \in \{1, 2, 3\}\) representing the other hands. Then \(P(E_i) = \binom{31}{13} / \binom{39}{13}\) for each \(i\). Let \(E\) be the event that one of the other hands is void in spades. Then \(E = \bigcup_{i=1}^{3} E_i\). By inclusion/exclusion

\[
P(E) = \binom{3}{1} \binom{31}{13} - \binom{3}{2} \binom{31}{13} \binom{18}{13} + \binom{3}{3} \cdot 0
\]

since for two hands to be void in spades you can pick the first hand from the 31 non-spades and the second hand from the remaining 18 non-spades, and then the third hand is left with 5 non-spades and 8 spades.

That is: The probability that there is at least one hand void in spades given you have 13 cards of which 5 are spades is \(\binom{4}{1} \binom{31}{13} - \binom{4}{2} \binom{31}{13} \binom{18}{13} / \binom{39}{13}\).