

In writing up solutions you should justify your conclusions by giving a clear chain of reasoning.

1. Six observations of a normal population of mean μ and variance σ^2 yield the values 20, 24, 25, 21, 23, and 25. In each case below, determine whether or not you can reject the null hypothesis at the 0.05 level of significance. (Note that (iii) and (iv) refer to σ , not σ^2 .)

- (i) $H_0: \mu = 25$ (ii) $H_0: \mu \leq 21$ (iii) $H_0: \sigma = 5$ (iv) $H_0: \sigma \leq 1.35$
 $H_1: \mu \neq 25$ $H_1: \mu > 21$ $H_1: \sigma \neq 5$ $H_1: \sigma > 1.35$

2. Consider two simple hypotheses about the parameter θ of some population: the null hypothesis H_0 is $\theta = \theta_0$, the alternative hypothesis H_1 is $\theta = \theta_1$.

(a) Suppose that C is the critical region for some test of these hypotheses, based on a sample of size n . Define carefully, in terms of C , θ_0 , and θ_1 , (i) the *size* of the critical region, (ii) the probability α of a type I error, (iii) the probability β of a type II error, and (iv) the *power* of the test.

(b) What would it mean to say that C is a *most powerful critical region* of size 0.05?

(c) State carefully the Neyman-Pearson Lemma.

3. A statistician studying a Bernoulli population with parameter θ decides to test the null hypothesis $\theta \leq 1/3$ against the alternative $\theta > 1/3$ using the test the critical region $X \geq 2$, where X is the number of successes in three trials. Find the size and power function of this test.

4. Samples are taken from two normal populations with variances σ_1^2 and σ_2^2 ; the first population yields values 10, 12, 15, 13, and 10, while the second yields values 20, 18, 17, 17.

(a) Test the null hypothesis $\sigma_2^2 \geq 4\sigma_1^2$ at the 5% level.

(b) Give a 95% confidence interval for σ_2^2/σ_1^2 .

5. Random samples of size 10 from two independent normal populations, each having variance 5, yield $\bar{x}_1 = 4$ and $\bar{x}_2 = 4.9$.

(a) Test the null hypothesis $\mu_1 \geq \mu_2$ at the 5% level.

(b) If in fact $\mu_1 = \mu_2 - 1$, what is the probability of a type II error with the test you used in (a)?

6. A random sample of size n is drawn from a beta distribution for which it is known that $\alpha = 1$. Find the estimators of β obtained by the method of moments and the method of maximum likelihood.

7. A certain population has density uniformly distributed on the interval $[0, \theta]$. We wish to test the null hypothesis $\theta = \theta_0$ against the alternative $\theta \neq \theta_0$ using a random sample X_1, \dots, X_n and a test constructed by the likelihood ratio method.

(a) Show that the value of the likelihood ratio statistic $\Lambda(X_1, \dots, X_n)$ is given by

$$\lambda = \begin{cases} \left(\frac{y_n}{\theta_0}\right)^n, & \text{if } Y_n \leq \theta_0, \\ 0, & \text{if } y_n > \theta_0, \end{cases}$$

where y_n is the largest of the sample values.

(b) Show that for some number k the resulting test has the form

“Reject H_0 if $y_n > \theta_0$ or $y_n < k$.”

Hint: graph λ as a function of y_n .

(c) Find k so that this test has size α .