**Problems on the pigeonhole principle (Version: September 16)**

1. Let $A$ be a subset of integers of size $n$. Prove that there is a nonempty subset of $A$ whose sum is divisible by $n$.

2. Let $A$ be a subset of size $n+1$ consisting of positive integers in the range 1 to $2n$. Prove that there must be distinct elements $a, b$ of $A$ such that $a$ is a divisor of $b$

3. Let $A$ be a finite subset of positive integers of size $n$ Let $a_1, a_2, \ldots, a_t$ be a sequence of integers each belonging to $A$. Prove that if $t \geq 2^n$ then there are integers $j, k$ satisfying $1 \leq j \leq k \leq n$ such that $\prod_{i=j}^{k} a_i$ is a perfect square.

4. For any positive integer $n$, if $S$ is a set of $2^n + 1$ points $\mathbb{R}^n$ with integer coordinates. Then there exists two of the points such that the midpint of the segment between them has all integer coordinates.

5. (Problem 2.6.10 from the book) Let $X$ be a real number and $n$ a positive integer. Prove that at least one of the numbers $X, 2X, ..., nX$ is within $1/(n+1)$ of an integer.

6. Let $M$ be a matrix of real numbers, with each row in nondecreasing order. Suppose we sort each column into nondecreasing order. Prove that the rows are still in nondecreasing order.

7. Let $B$ be a subset of $\{-1, 1\}^n$ (the set of points in $\mathbb{R}^n$ with all coordinates equal to $-1$ or $+1$. If $|B| > 2^{n+1}/n$, prove that $B$ contains a set of three points that are the vertices of an equalatoral triangle.

8. Let $m, n$ be positive integers. Suppose $x_1, \ldots, x_m$ are positive integers between 1 and $n$ and $y_1, \ldots, y_n$ are positive integers between 1 and $m$. Prove that there is a nonempty subsequence of $x_1, \ldots, x_m$ and a nonempty subsequence of $y_1, \ldots, y_n$ that have the same sum.

9. (Somewhat difficult) Let $A, B$ be integer 2 by 2 matrices. Suppose that each of the matrices $A, A+B, A+2B, A+3B, A+4B$ has the property that it is invertible and its inverse has integer entries. Prove that $A+5B$ has the same property.