PROBLEM SOLVING SEMINAR, FALL 2008

Suggested problems from book: 3.1.10, 3.1.16, 3.2.11, 3.2.17, 3.3.14.

Problem 1: Prove that every integer is a divisor of infinitely many Fibonacci numbers.

Problem 2: Show that the product of the side lengths of any right triangle with integer side lengths is divisible by 60.

Problem 3: For $n \in \mathbb{N}$, let $H(n) = 1 + 1/2 + 1/3 + \cdots + 1/n$. Prove that for $n \geq 2$, $H(n)$ is not an integer.

Problem 4. [Mike Zieve]: Let $f(X) \in \mathbb{Z}[X]$ be a polynomial with integer coefficients. Let $a_1, a_2, \ldots, a_n \in \mathbb{Z}$ be integers such that $f(a_i) = a_{i+1}$ for $1 \leq i < n$, and $f(a_n) = a_1$. Prove that $n \leq 2$.