

640:501 Theory of Functions of a Real Variable, Fall 2003

Syllabus by topics and section of text

1. Functions of bounded variation. Text: 2.1-2.2
2. Cantor sets and Cantor functions. Text: 3.1
3. The Riemann and Riemann-Stieltjes integrals. Text: 1.7,2.3,2.4
4. The drawbacks of the Riemann-Stieltjes integral. Course notes
Limits and the Riemann integral.
Fourier series and completion of the L^2 -norm.
5. Outline of the Lebesgue approach to integration.
Measure spaces, measures, and integration.
6. Algebras and σ -algebras of sets. Course notes
Borel sets
Monotone class and π - λ system theorems.
7. Construction of measures. Text: 10.1, pp 161-163, Chapter 11
Carathéodory's theorem for constructing a measure space from an outer measure.
Criterion for extension of a finitely additive measure to a measure.
Application to Lebesgue and Lebesgue-Stieltjes measures. Text: 3.1
Application to a coin-tossing model. Course notes
Hausdorff measure.
8. Regularity of Lebesgue and Lebesgue Stieltjes measures. Text: 3.2-3.5
Regular Borel measures.
The relationship between Lebesgue measurable sets and Borel measurable sets.
Completion of measures.
9. Measurable functions. Text 4.1
10. The integral of measurable functions on a measure space. Section 10.2, Chapter 5
Monotone convergence theorem, Fatou's lemma, dominated convergence theorem.
Measurable transformations and transformations of integrals.
Characterization of Riemann integrability.
11. Types of convergence.
Almost sure convergence, convergence in measure, convergence in L^p -norm.
Uniform integrability.
12. Product measures. Text: Chapter 6
Fubini-Tonelli theorem.
13. Regularity of measurable functions; Lusin and Egoroff theorems. Text: 4.3
14. Absolute continuity of measures and the Radon-Nikodym theorem. Text: 10.3
15. Differentiation of measures. Text: Chapter 7
Vitali's covering theorem.
Hardy's maximal function.
The Lebesgue differentiability theorem
Application to differentiability of bounded variation functions

Absolutely continuous functions.

16. If time remains, we will do some lectures on special topics. Examples: The ergodic theorem, construction of Haar measures.