

LECTURE NOTES ON
ORDINARY DIFFERENTIAL EQUATIONS

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TABLE OF CONTENTS

I. INTRODUCTION	1
II. EXISTENCE, UNIQUENESS, AND ALL THAT	6
2.1 Existence and Uniqueness under a Lipschitz condition	7
2.2 Existence without a Lipschitz condition	12
2.3 Continuity in the initial conditions	15
2.4 Dependence of the solution on parameters	18
2.5 Differentiability of the solution in the initial conditions	20
2.6 Extension of solutions	26
III. LINEAR DIFFERENTIAL EQUATIONS	29
3.1 General properties	29
3.2 Linear Equations with Constant Coefficients	33
3.3 Computation of Jordan Canonical Form	42
3.4 Linear Equations with Periodic Coefficients: Floquet Theory	45
IV. AUTONOMOUS SYSTEMS	48
4.1 Orbits and asymptotic limit sets	48
4.2 Two dimensional autonomous systems and linearization	52
4.3 The Poincaré-Bendixson Theorem	63
V. STABILITY	68
5.1 Linearized stability analysis of constant solutions	68
5.2 Stability of periodic solutions of non-autonomous systems	74
5.3 Stability of periodic solutions of autonomous systems	76
5.4 The Second Method of Lyapunov	82
5.5 Stable and Unstable Manifolds	87
Appendix to Chapter V	92

PREFACE

These are rough notes based on lectures given at Rutgers University in 1988, 1989, and 1995. The book *Differential Equations: Introduction and Qualitative Theory*, by Jane Cronin, was used as a text for the first two of these years, and this influenced the order of topics here, but the treatment of most topics is quite different.

I give below a list of sources consulted during the preparation of the notes. Comments are included which may guide the student in her reading.

I would appreciate being informed of any errors, minor or substantial.

Eugene R. Speer

REFERENCES

[1] Arnold, V. I., *Ordinary Differential Equations*. Trans. Richard A. Silverman. Cambridge, Massachusetts: MIT Press, 1973.

A book with a very geometric approach. The emphasis is on linear and autonomous systems.

[2] Birkhoff, Garrett, and Gian-Carlo Rota, *Ordinary Differential Equations*. 3rd ed. New York: John Wiley and Sons, 1978.

Covers material from a standard American undergraduate o.d.e. course but also more advanced material overlapping these notes.

[3] Boyce, William E., and DiPrima, Richard C., *Elementary Differential Equations*. 4th ed. New York: John Wiley and Sons, 1986.

A excellent undergraduate text, mostly on methods of explicit solution. Possibly valuable for background reading. Any edition would do.

[4] Coddington, Earl A., and Norman Levinson, *Theory of Ordinary Differential Equations*. New York: McGraw-Hill, 1955; rpt. Malabar, Florida: Robert E. Krieger Publishing Company, 1987.

A very comprehensive text; more “classical” in spirit than some others here. This and [7] are probably the best books for extensive reference.

[5] Cronin, Jane, *Differential Equations: Introduction and Qualitative Theory*. New York: Marcel Dekker, Inc., 1980.

A nice book with topics very similar to those covered here.

[6] Hale, Jack K., *Ordinary Differential Equations*. New York: John Wiley and Sons, 1969; rpt. Malabar, Florida: Robert E. Krieger Publishing Company, 1980.

Mathematically sophisticated presentation; modern material.

[7] Hartman, Philip, *Ordinary Differential Equations*. 2nd ed. New York, John Wiley and Sons, 1973; rpt. Boston, Birkhäuser, 1982.

Very, very comprehensive; lots of good material. This and [4] are probably the best books for extensive reference.

[8] Hirsch, Morris W. and Stephen Smale, *Differential Equations, Dynamical Systems, and Linear Algebra*. Orlando, Florida: Academic Press, 1974.

Emphasis similar to that of [1]; lots of material on linear systems. Includes applications.

[9] Lefschetz, Solomon, *Differential Equations: Geometric Theory*. 2nd ed. Interscience, 1963; rpt. New York: Dover Publications, 1977.

This book has most of our topics; its style is a little old fashioned.

[10] Petrovski, I.G., *Ordinary Differential Equations*. Trans. and ed. Richard A. Silverman. Prentice Hall, 1966; rpt. Dover Publications, 1973.

A nice, short book. It is quite cheap and would be convenient for supplementary reading.