# 642:515 ODE Problem Set 0

*Note: these are "review" problems, actually taken from an undergraduate course.*

**1.** Solve the following initial value problems:

a.

$$y' \;=\; -2ty \,+\, 4e^{-t^2}, \quad y(0) = 3\,.$$

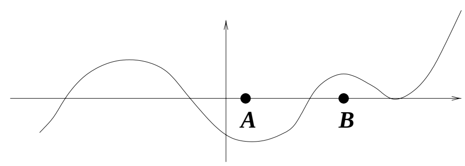
b.

$$y' \;=\; \frac{t(t^2+1)}{5y^4}\,, \;\; y(0) = 2\,.$$

c.

$$y' \;=\; \frac{y^3+1}{3y^2}\,, \;\; y(0) = 2\,.$$

**2.** This is the graph of a function $f(y)$.




Sketch the phase line for the differential equation $dy/dt = f(y)$, labeling all the equilibrium points and writing next to each of them if they are a source, sink, or node. What happens to solutions starting at $A$? At $B$?

**3.** Match each of the word descriptions to one of the differential equations.
Descriptions:

- The temperature shown by a thermometer which has been immersed in a container with hot water.
- The velocity at which a parachute jumper is falling. The air drag is assumed to be proportional to the square of the velocity. (Positive direction is downwards.)
- The rate of change of the volume of a raindrop, which evaporates at a rate proportional to its surface area.
- A drug was originally administered to a patient. The equation describes the amount of the drug in the patient's blood at time $t$. The patient's body eliminates the drug from the blood at a rate which is proportional to how much of the drug is present in the blood at any given time.
- As above, but there is also an IV which keeps delivering more of the drug, at a constant rate per unit of time.

Equations (all constants are positive):

| | | | | |
|---|---|---|---|---|
| **1**. $y' = -ky + cy^2$ | **2**. $y' = -ky - cy^2$ | **3**. $y' = ky + cy^2$ | **4**. $y' = -k(y-c)$ | **5**. $y' = ky - cy^2$ |
| **6**. $y' = -ky - c$ | **7**. $y' = -ky$ | **8**. $y' = k - cy^2$ | **9**. $y' = k + cy^2$ | **10**. $y' = -ky + c$ |
| **11**. $y' = ky^{1/3} + c$ | **12**. $y' = ky^{2/3} + c$ | **13**. $y' = -ky^{2/3}$ | **14**. $y' = ky^{2/3}$ | **15**. $y' = -ky^{2/3} + c$ |

*More than one equation may match a description.*

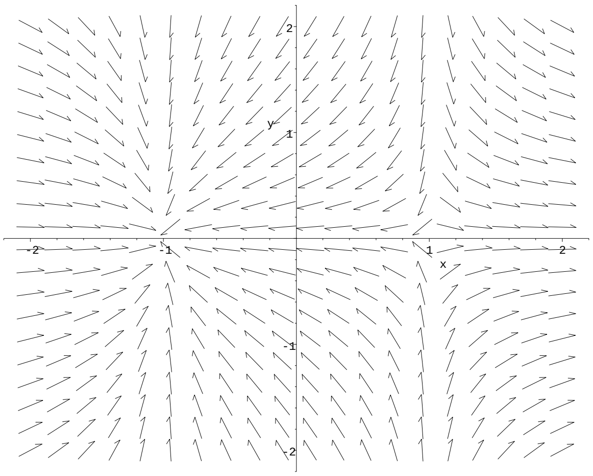
**4.** (linear equation) Find the general solution of:

$$y' \;=\; 3y + te^{3t}\,.$$

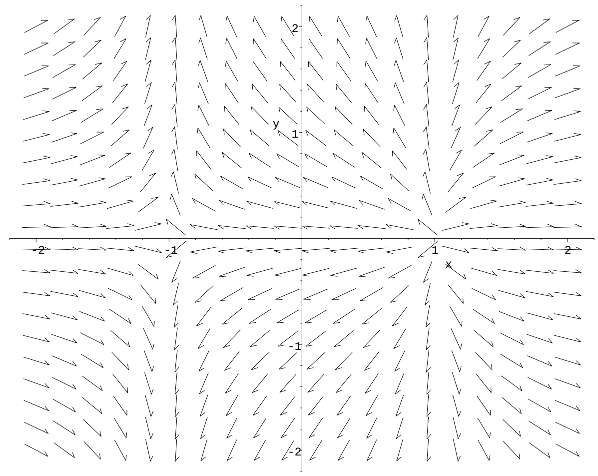
**5.** Consider these differential equations:

| | | | |
|---|---|---|---|
| $\frac{dx}{dt} = x^2 - 1$ <br> $\frac{dy}{dt} = -y$ | $\frac{dx}{dt} = -x^2 - 1$ <br> $\frac{dy}{dt} = y$ | $\frac{dx}{dt} = x^2 - 1$ <br> $\frac{dy}{dt} = y$ | $\frac{dx}{dt} = -x^2 - 1$ <br> $\frac{dy}{dt} = -y$ |
| (**A**) | (**B**) | (**C**) | (**D**) |

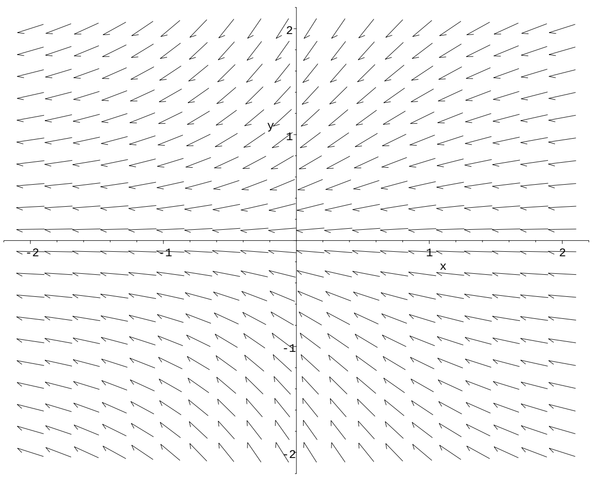
and these direction fields:



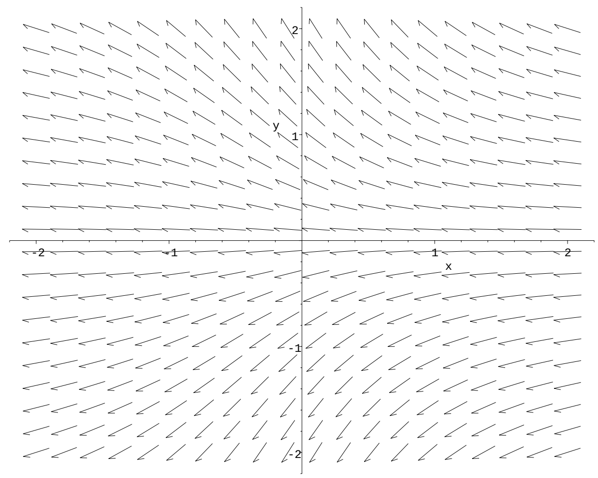
I



II



III



IV

**Match** the direction fields and equations: A: ☐ B: ☐ C: ☐ D: ☐

**Also:**

- Draw, in each of the four pictures I-IV, the solution $(x(t), y(t))$, for $t > 0$, with the initial conditions $x(0) = 0.9$ and $y(0) = 0.1$.
- Answer: for which (there may be one or more) of the pictures I-IV, does the solution $(x(t), y(t))$, for $t > 0$, with the initial conditions $x(0) = 0.9$ and $y(0) = 0.1$, have the property that, as $t \to \infty$, the trajectory $(x(t), y(t))$ approaches some (finite) point?