Mathematics 535 | Algebraic Geometry | Fall 2008

Problem Set 3B (Last revised 10/3/2008)

1.12 Show that any finite set of points on the twisted cubic is in general position.

1.13 Let p_i be 7 distinct points on a twisted cubic C. Show that the zero locus of all quadratic polynomials which vanish at all points p_i is precisely C.

1.15 Let p_i be kd+1 distinct points on a rational normal curve C in $\mathbf{P}^d$. Show that any polynomial F of degree k vanishing on the p_i also vanishes on C. Show that exercise 1.5 is sharp.

2.8 Show that the image $Y = \nu_d(X) \subset \mathbf{P}^N$ of a projective variety $X \subset \mathbf{P}^n$ is isomorphic to X via the Veronese map.

2.9 Use 2.8 to show that any projective variety is isomorphic to the intersection of a Veronese variety and a linear subspace, and hence is isomorphic to an intersection of quadrics.

2.10 Let $Y = \nu_d(X) \subset \mathbf{P}^N$ be the Veronese image of a projective variety $X \subset \mathbf{P}^n$. What is the relation of the homogeneou s coordinate rings? Show that the homogeneous coordinate ring is not an isomorphis m invariant of a projective variety, and construct varieties in a projective space that are isomorphic but not projectively equivalent.

2.13 Show that the twisted cubic in $\mathbf{P}^3$ is the intersection of the Segre threefold with a three plane in $\mathbf{P}^5$.