Problem Set 4A (Last revised 10/7/2008)

3.2 Let $\Psi, \Lambda \subset \mathbf{P}^n$ be complementary linear subspaces and let $X \subset \Psi, Y \subset \Lambda$ be subvarieties. Show that the union of all lines joining points of X to points of Y is a variety.

3.7 Show that if ℓ is any line in $\mathbf{P}^n$ not meeting a subvariety X then there exist homogeneous polynomials $F, G \in I(X)$ with no common zeros on ℓ.

3.8 Find the equations of the projection of the twisted cubic curve from the point $[1,0,0,1]$ and from $[0,1,0,0]$. Show that any projection of the twisted cubic from a point not on the cubic is projectively equivalent to one of these two.

5.3 Show that the homogeneous quadratic polynomials

$$F_{i,i}(Z) = Z_i^2 - Z_{i-1}Z_{i+1}$$

and

$$F_{i,i+1}(Z) = Z_iZ_{i+1} - Z_{i-1}Z_{i+2}$$

for $i = 1, \ldots, d-1$ generate the ideal of the rational normal curve locally but do not generate the homogeneous ideal $I(C)$.

5.4 Show that the polynomials $F_{i,j}(Z) = Z_iZ_j - Z_{i-1}Z_{j+1}$ for $1 \leq i \leq j \leq d-1$ generate the homogeneous ideal of the rational normal curve. Show that the equations we saw earlier for the Veronese and Segre varieties generate their homogeneous ideals.