Problem Set 6B (Last revised 11/30/2008)

11.9 Let $\Gamma \subset \mathbf{P}^n$ be a general collection of d points. It is known that Γ is a complete intersection if and only if $n = 1, d = 1, 2$ or $d = 4, n = 2$. Show this for $n = 2, d \leq 7$.

11.10 Let $\Gamma \subset \mathbf{P}^2$ be a general collection of d points. Show that Γ is a set-theoretic complete intersection.

11.25 Show that if $X \subset \mathbf{P}^n$ is an irreducible curve then the chordal variety $S(X)$ is three-dimensional unless X is contained in a plane.

11.29 Let M be the projective space of $m \times n$ matrices, and let $M_k \subset M$ be the subvariety of matrices of rank at most k. Take k so that $2k < \min(m, n)$. Show that the chordal variety $S(M_k)$ is equal to the subvariety $M_{2k} \subset M$ of matrices of rank at most $2k$. Indeed. show that for a map $A : K^m \to K^n$ of rank $2k$ and any pair of complementary k-dimensiaaonal subspaces $\Lambda, \Xi \subset$ image$M(A)$ the composition of A with projections of Λ and Ξ give an expression of A as a sum of two matrices of rank k. Deduce that in the case $\mathbf{P}^n = M, X = M_k$ the fiber of the incidence correspondence Σ in 11.22 over a general point $S(X)$ has dimension $2k^2$ and hence that $\dim(S(M_k)) \leq 2\dim(M_k) + 1 - 2k^2$.