

## Problem Set 7A (Last revised 12/1/2008)

- 13.6 Find the Hilbert function of the Segre variety  $\Sigma_{n,m} = \sigma(\mathbf{P}^n \times \mathbf{P}^m) \subset \mathbf{P}^{nm+n+m}$  and verify that the dimension is  $n + m$ .
- 13.8 Determine the arithmetic genus of (i) a pair of skew lines in  $\mathbf{P}^3$  (ii) a pair of incident lines in either  $\mathbf{P}^2$  or  $\mathbf{P}^3$  (iii) three concurrent but not coplanar lines in  $\mathbf{P}^3$  and (iv) three concurrent coplanar lines in either  $\mathbf{P}^2$  or  $\mathbf{P}^3$ .
- 13.9 Consider a plane curve  $X \subset \mathbf{P}^2$  of degree  $d$  and its image  $Y = \nu_2(X) \subset \mathbf{P}^5$  under the quadratic Veronese map. Compare the Hilbert polynomials of the two and observe in particular that the arithmetic genus is the same.
- 13.17 Prove a weak form of the Bezout theorem in  $\mathbf{P}^2$ : if  $F, G$  are polynomials of degree  $d, e$  on  $\mathbf{P}^2$  without common factors such that  $F, G$  generate the ideal of their intersection, then the intersection consists of  $d \cdot e$  points. Similarly show that if  $\Gamma \subset \mathbf{P}^3$  is a complete intersection of surfaces of degrees  $d, e, f$  then  $\Gamma$  consists of  $d \cdot e \cdot f$  points.
- 13.18 Find the Hilbert polynomial of a complete intersection in  $\mathbf{P}^3$  of surfaces of degrees  $d, e$ . What is the arithmetic genus of this complete intersection?