Problem Set 7B (Last revised 12/1/2008)

18.2 Assuming that the characteristic of K is 0, use exercises 7.16 and 11.44 to show that part (iii) of Definition 18.1 is well-defined, and use this to show that parts (i) and (ii) are also well-defined.

18.7 Let $X \subset \mathbf{P}^n$ be any subvariety of pure dimension of degree 1. Show that X is a linear subspace of $\mathbf{P}^n$.

18.11 Give a proof of the nondegeneracy of the general hyperplane section of the variety X without invoking the notion of degree or Bezout's theorem as follows. First show that if the general hyperplane section of X spans a k-plane we have a rational map $\phi : \mathbf{P}^{n*} - - \to \mathbf{G}(k, n)$ defined by sending a general hyperplane $H \in \mathbf{P}^{n*}$ to the span of $H \cap X$. Next use the fact that the universal hyperplane section of X is irreducible to deduce that for any $H \in \mathbf{P}^{n*}$ and any point $\Lambda \in \phi(H)$ the hyperplane section $H \cap X$ lies on the k-plane Λ. It follows that if the general hyperplane section is degenerate then all are. But any n independent points of X will span a hyperplane H with $H \cap X$ nondegenerate.

18.14 Let $Y \subset \mathbf{P}^n$ be a variety of dimension k and degree e, and $Z = \nu_d(Y)$ its image in $\mathbf{P}^N$ under the Veronese map. Use the various degree computing techniques to compute the degree of Z.

18.18 Let K have characteristic 0. Let $X, Y \subset \mathbf{P}^n$ be disjoint subvarieties of dimensions k and l, and $\Gamma \subset \mathbf{P}^n$ a general plane of dimension $n-k-l-1$. Show that the projections $\pi_\Lambda(X)$ and $\pi_\Gamma(X)$ intersect transversely.