

Add to Exercises 1.3.7:

7. Let $X, Y \in M_n(\mathbb{R})$. Use Lemma 1.3.6 to prove that there exists an $\epsilon > 0$ and a constant $C > 0$ so that the following holds:

(a) If $\|X\| + \|Y\| < \epsilon$ then

$$\exp X \exp Y \exp -X = \exp (Y + [X, Y] + Q(X, Y)),$$

with $Q(X, Y) \in M_n(\mathbb{R})$ and $\|Q(X, Y)\| \leq C(\|X\| + \|Y\|)^3$.

(b) If $\|X\| + \|Y\| < \epsilon$ then

$$\exp X \exp Y \exp(-X) \exp(-Y) = \exp ([X, Y] + P(X, Y)),$$

with $P(X, Y) \in M_n(\mathbb{R})$ and $\|P(X, Y)\| \leq C(\|X\| + \|Y\|)^3$.