Mathematics 551 **Algebra** **Fall 2006**

Week 6 Further topics in group theory, Introduction to rings
Sections 3.1–3.2

1. Show that a group generated by r elements has at most $n!^r$ subgroups of index no larger than n by relating subgroups of index n to group actions on a set of cardinality n.
2. (Frattini) Let G be a finite group.
   a) Let P be a Sylow subgroup of a normal subgroup N of G. Show that conjugation by elements of G permutes the Sylow subgroups of N and that $G = N_G(P)N$.
   b) Let the Frattini subgroup $Frat(G)$ be the intersection of all maximal proper subgroups of G. Show that $Frat(G)$ is mapped to itself by any automorphism of G (in particular it is normal) and that the Sylow subgroups of $Frat(G)$ are normal in G
   c) Let A be a finite abelian group. Compute the Frattini subgroup of A in terms of invariant factors or elementary divisors of A.
3. Hungerford 3.1.6
4. Hungerford 3.1.3
5. Hungerford 3.2.1
6. Hungerford 3.2.8
7. Hungerford 3.2.18
8. Hungerford 3.2.23