Homework 3

1. Go to http://www.math.rice.edu/∼dfield/dfpp.html, click on PPLANE 2002.2 and use this software to visualize the phase plane of a 2D system of ODE's. Consider the system from Homework 2,

$$\dot{y} = ry\left(1 - \frac{y}{k}\right) - ay - pyz, \tag{1}$$
$$\dot{z} = \frac{cyz}{1+\epsilon y} - qyz - bz, \tag{2}$$

   and see how the phase portrait changes as you change the parameter r. Suggested values of other parameters are $a = 3$, $\epsilon = 10$, $k = 5$, $c = 12$, $p = 1$, $q = 0.01$, $b = 0.3$. Calculate the values of r that separate different regimes (see Homework 2) and draw phase diagrams for each of the regime. Zoom by pressing shift and clicking on the graph. Sketch what you see.

2. Use the same software to observe Hopf bifurcation. Use the system

$$\begin{aligned}\dot{x} &= \mu x + y - (x+y)(x^2+y^2),\\ \dot{y} &= -x + \mu y - (x+y)(x^2+y^2).\end{aligned}$$

   Sketch the phase portrait on both sides of the bifurcation.

3. Find a general solution by variation of parameters,

$$y'' - 2y' + y = e^x/x^3 + 1.$$

4. Solve the initial value problem,

$$\begin{aligned}\dot{y}_1 &= 2y_1 + 6y_2 + e^{-t},\\ \dot{y}_2 &= y_1 - 2y_2,\\ y_1(0) &= 0, \quad y_2(0) = 1.\end{aligned}$$

   First, write the matrix in the form $A = S\Lambda S^{-1}$, calculate e^{At} and go from there.