

Homework 4

1. Solve by Laplace transform,

$$y'' + 9y = g(t),$$

where $g(t) = \sin t$ for $0 \leq t \leq \pi$ and $g(t) = 0$ for $t > \pi$.

2. Find the complex Fourier series of the function $f(x)$ which is equal to $f(x) = x^2$ for $-\pi \leq x \leq \pi$ and has the period 2π .
3. Find the Fourier transform of the function $F(x)$ which is equal $F(x) = e^{\lambda x}$ for $-\infty < x \leq 1$, $F(x) = e^{\lambda}$ for $1 \leq x \leq 2$ and $F(x) = 0$ for $x > 2$.
4. Go to <http://www.falstad.com/fourier>, and use this software to visualize the Fourier series of a periodic function. Start by clicking on “sawtooth” on the right. This is the function $f(x)$ which equals $f(x) = x$ for $-\pi \leq x \leq \pi$, and has period 2π . What are the Fourier coefficients for this function? Plot the size of each sine (cosine) Fourier coefficient as the function of its sequential number. Set the “Number of terms” to be 1, 5, 10. Sketch the graph of the resulting series.
5. With the same applet, click on “Sine”. What are Fourier coefficients of this function? Why doesn't the picture change as you vary the number of terms?