

## Homework 5

1. Find the Laplace transform of  $\sin \pi t$ .
2. Solve by Laplace transform,

$$y'' + 3y' - 10 = t, \quad y(0) = 1, \quad y'(0) = 2.$$

3. Find the Fourier series of the function  $f(x) = \cos 2x$ .
4. Solve the diffusion equation for a half-line,

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad \frac{\partial u}{\partial x} \Big|_{x=0} = 0, \quad \lim_{x \rightarrow \infty} u(x, t) = 0, \quad u(x, 0) = \frac{1}{x^2 + 1}.$$

Are you using the Sine- or Cosine transform? Why? Leave the solution in the form of an integral.

5. Solve the diffusion equation on an interval,  $0 \leq x \leq 10$ ,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad u(0, t) = 0, \quad u(10, t) = 0, \quad u(x, 0) = e^{-10|x-5|}.$$

Plot the initial condition.

Then go to <http://www.math.ubc.ca/~feldman/demos/demo7.html> and use the applet to solve the equation numerically. Sketch what you see.

6. Go to <http://lewis.eeb.uconn.edu/lewishome/applets/Diffusion/diffusion.html> and observe the motion of the particles. How is this related to the diffusion equation? What (collective) characteristic of the particles is described by the diffusion equation? (What is the “ $u(x, t)$ ”?) What are the boundary conditions in this case (is it a condition on the function itself or on its derivative?) What is the long-term behavior of this function? Does it go to zero? Why?
7. How many different samples of 4 objects can be drawn from a lot of 50 objects?
8. A cage contains 100 mice, 3 of which are male. What is the probability that the three male mice will be included if 10 mice are randomly selected?

9. Calculate the mean and the variance of a random variable whose probability function is

$$f(x) = k \binom{3}{x},$$

and  $x = 0, 1, 2, 3$ . What is the value of  $k$ ? (Hint: what is  $\sum_{i=0}^3 f(x_i)$ ?)

10. Calculate the mean and the variance of a random variable whose probability density function is  $f(x) = kx$ ,  $0 \leq x \leq 2$ . What is the value of  $k$ ?