Homework 6

1. Write down the probability density function for the exponential distribution. Calculate the mean, the second moment and the variance. Show your steps.

2. If $f(x)$ is a continuous density distribution, then the quantity

$$M(t) \equiv \langle e^{xt} \rangle = \int f(x) e^{xt} \, dt$$

is the continuous analog of the discrete moment generating function. Compute $M(t)$ for the distribution in problem 1. Using derivatives of $M(t)$, calculate the mean and the second moment of x.

3. Suppose that the discrete binomial distribution, f_k, with $k = 0, 1, \ldots, n$ is given by

$$f_k = \left(\begin{array}{c} n \\ k \end{array} \right) p^k (1-p)^{n-k}.$$

What is the mean of this distribution? What is the standard deviation? Write down the Normal distribution, $f(x)$, which approximates this binomial distribution (for large values of n). (Hint: this distribution will have the same mean and the same variance). Plot both distributions on the same graph for parameters $p = 0.3$, $n = 10$ and then for $p = 0.3$, $n = 100$. Mark the mean. For both graphs, measure (approximately) the width and demonstrate that it is related to the standard deviation.

4. Go to http://www.uno.it/utenti/tetractys/askapplets/bernoulli2Gauss1.htm Observe for some time. The fallen particles follow a certain shape. Draw this shape. It can be described by a Normal distribution. Prove this. (Hint: at each discrete moment of time, the falling particles move left or right, by a fixed increment Δx. They move randomly, with the probability for each particle to move left or right being 1/2. Each particle is only allowed to move left/right a fixed number of times, say K times.)

   Next, Let us now suppose that we let the particles fall and move left/right for longer (K is larger). What will be the resulting shape (what is the difference)?

5. Go to http://www.math.uah.edu/psol/applets/MarkovChainExperiment.html Set n=5 on the sliding bar. Click on the small blue table underneath "Play" and choose the transition matrix to be "RW", which is a symmetric random walk with reflecting boundaries. Run the Markov process until it seems stationary. Draw the distribution that you see. What does the height of columns on this bar-chart tell us? Calculate the stationary probability distribution for this matrix. Why is it not the same as the bar chart? Can you give a simple explanation of why the stationary probability distribution is the way it is?