Part III: playing with calculus on *Maple*

*We will no longer indicate the return RET at the end of each command line in Maple*

The basic calculus commands do differentiation and integration. Let’s try them.

\[
\text{diff}(3x^7 - 22.1x^2, x) \quad \text{int}(x \cdot \sqrt{x+2}, x)
\]

This integration is not easy. *Maple* either substituted or it integrated by parts (it can do both). Typing the * explicitly forces *Maple* to recognize variables. I’ve done some “experiments” and the results suggest that writing *’s rather than having *Maple* guess leads to fewer misunderstandings.

*Maple* likely knows all the functions you do, and many others. The function \( \sin \left( x^2 \sqrt{x + 1} \right) \) and the calculus rules are known. Let’s assign this expression a name and then play.

\[
Q := \sin(x^2 \cdot \sqrt{x+1})
\]

\[
\text{diff}(Q, x)
\]

This should get the first derivative. How many ways can you think of to get the second derivative? First, immediately after the command and response above, type

\[
\text{diff}(\% , x)
\]

This will do it. For an independent computation, try the command line

\[
\text{diff}(Q, x, x)
\]

How about the tenth derivative? First type

\[
x^\$10
\]

to see what \$ means. Now you can find the tenth derivative of the function \( \sin \left( x^2 \sqrt{x + 1} \right) \). Realize (even if the tenth derivative of this function is needed) why people end commands with : (which turns off the output) rather than displaying the output. You may want the results of a computation, but you may not have the need or desire to actually look at it!

What is the coefficient of \( x^3 \) in the seventh derivative of \( \left( x^2 + \frac{1}{x^2} \right)^5 \)? First compute the indicated derivative. You’ll get a mess. Then have *Maple* massage the result algebraically so you can read off the answer. I’m an amateur and first tried \text{expand}(\%) and I also tried \text{simplify}(\%) and the results were different. Another way is to first \text{expand} \( \left( x^2 + \frac{1}{x^2} \right)^5 \) and then differentiate the result seven times. The answer should be the same!

Let’s look at integration a bit more closely. Define \( V \) to be \( e^{\sin x} \):

\[
V := \exp(\sin(x))
\]

Now let’s integrate it. First (try this carefully!) type

\[
\text{int}(V, w)
\]

and explain the result to yourself. Remember, a program will do what you tell it to do! Now try

\[
\text{int}(V, x)
\]

and you may wait a bit and then have something else to explain. *Maple* has the usual integration algorithms and many, many other antidifferentiation tricks. An answer like this is a fairly good hint that it “can’t be done”: that is, the antiderivative can’t be expressed in terms of familiar functions with familiar ways of combining them, including sum, product, composition, . . . even using the rather large collection of functions known to *Maple*. Such results can be proved!

OVER
We can also compute definite integrals. For example,

\[
\text{int}(x^3, x=1/7..b)
\]

computes \( \int_{1/7}^{b} x^3 \, dx \) (if you want it!). \textit{Maple} indicates a range (for integration and for other purposes) by the notation \texttt{variable=lower\_limit..upper\_limit}.

Remember \( V \)? Be sure \textit{Maple} does (check by typing \( V \)) and compute \( \int_{0}^{1} e^{\sin x} \, dx \) by typing

\[
\text{int}(V, x=0..1)
\]

and consider the result. Disappointment is decreased if we follow that answer with

\[
\text{evalf}(\%);
\]

You can evaluate \( V \) itself with a command like

\[
\text{subs}({x=3}, V)
\]

followed by

\[
\text{evalf}(\%)
\]

If you’re not scared, you could have done this all together by typing

\[
\text{evalf(subs}({x=3}, V))
\]

but sometimes I get confused by the matching required (in count and type) of all the parentheses. We could similarly evaluate a derivative of \( V \) by differentiating with \texttt{diff}, substituting, and then \texttt{evaluating}. Or we could define our own functions. Initially the syntax may seem burdensome, but here is a simple example.

\[
N:=x\rightarrow \arctan(x^3)
\]

So: call \( N \) the function which takes the input \( x \) and assigns to it \( \arctan(x^3) \). Try this

\[
N(2)\;N(5\times z)
\]

which shows that \textit{Maple} believes \( N \) is a function. Now type

\[
\text{diff}(N(x), x);
\]

That’s an \textit{expression} and not a function. There’s no way to “plug in” \( K + 3 \) easily in it (yes, we could use the \texttt{subs} command, but that’s cumbersome). So \textit{Maple} has another way to differentiate functions (such as \( N \)) rather than expressions (such as \( V \)). Try

\[
D(N);
\]

and view the result. Indeed: call it by a new name, say, \( M \):

\[
M:=D(N);
\]

Now evaluate \( M(3) \) and \( M(K + 3) \).

Please check the difference between

\[
\text{int}(N(x), x);
\]

and

\[
\text{int}(N,x);
\]

One of these has an answer that’s fun to me (because I didn’t have to compute it and can appreciate the work involved!) and the other seems silly: the function \( N \) hasn’t been told what to evaluate, so it can’t be antidifferentiated.

The difference between “expressions” like \( V \) and “functions” like \( N \) can be subtle. Expressions seem more static, while functions have a more dynamic aspect – substituting is built into their structure.

\textit{Maple} also knows about such calculus topics as limits, sums, and series. If you need to work with any of these, look at \texttt{help(limit)} and \texttt{help(sum)} and \texttt{help(series)}. I almost always look at the examples first. Don’t be afraid to experiment, and look at the \texttt{help} pages if the command you are investigating doesn’t do exactly what you want.

It’s time to \textit{graph}.