Math 252 — Fall 2002
Answers to Modeling Exercises

Descriptions
1. The rate of change of the population of a certain country, which depends on the birth and death rates as well as on the number of immigrants, who arrive at a constant rate into the country.
   \[ y' = ky + c \]
2. The rate of change of the population of a certain country, which depends on the birth and death rates, but there is a net emigration from the country (at a constant rate).
   \[ y' = ky - c \]
3. Fish in a certain area, which reproduce in proportion to the population, subject to limits imposed by the carrying capacity of the environment, and the population of which is also reduced by fishing which proceeds at a constant rate.
   \[ y' = ky(K - y) - c \]
4. The temperature of a building, when the outside temperature varies periodically (it goes down during the night, up during the day) and there is no heating or air-conditioning.
   \[ y' = -k(y - \sin t) \]
5. The temperature of a building, when the outside temperature varies periodically (it goes down during the night, up during the day) and heating is being applied at a constant rate.
   \[ y' = y' = -k(y - \sin t) + c \]
6. The temperature of a building, when the outside temperature is constant, and there is no heating or air-conditioning.
   \[ y' = -k(y - K) \]
7. The temperature of a building, when the outside temperature is constant, and heating is being applied at a constant rate.
   \[ y' = -k(y - K) + c \]
8. The amount of money in a savings account, when interest is compounded continuously, and also additional money is being added at a constant rate (the person always deposits a certain percentage of her paycheck).
   \[ y' = ky + c \]
9. The rate of change of the volume of a raindrop, which evaporates at a rate proportional to its surface area.
   \[ y' = -k^{2/3} \]
10. The rate of change of the volume of a raindrop, which evaporates at a rate proportional to its diameter.
    \[ y' = -k^{1/3} \]
11. The mass of a radioactive substance which is decaying (at a rate proportional to the amount present).
    \[ y' = -ky \]
12. The amount of chlorine in a swimming pool; chlorinated water is added at a fixed rate, the water in the pool is well-mixed, and water is being removed from the pool so that the total volume is constant.
    \[ y' = -ky + c \]
Equations (all constants are positive):

• \( y' = -ky \) \hspace{1cm} \text{Answer(s):} \hspace{1cm} 11

• \( y' = -ky^{1/3} \) \hspace{1cm} \text{Answer(s):} \hspace{1cm} 10

• \( y' = ky(K - y) \) \hspace{1cm} \text{Answer(s):} \hspace{1cm}

• \( y' = ky(K - y) - c \) \hspace{1cm} \text{Answer(s):} \hspace{1cm} 3

• \( y' = -k(y - \sin t) \) \hspace{1cm} \text{Answer(s):} \hspace{1cm} 4

• \( y' = -k(y - K) + c \) \hspace{1cm} \text{Answer(s):} \hspace{1cm} 7

• \( y' = -k(y - K) \) \hspace{1cm} \text{Answer(s):} \hspace{1cm} 6

• \( y' = ky + c \) \hspace{1cm} \text{Answer(s):} \hspace{1cm} 1, 8

• \( y' = ky - c \) \hspace{1cm} \text{Answer(s):} \hspace{1cm} 2