ORAL QUALIFYING EXAM SYLLABUS
SPRING 2017
KEITH FRANKSTON

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Major topic: Combinatorics
Minor topic: Symmetric Functions

1. COMBINATORICS AND GRAPH THEORY

1.1. Combinatorics.
- Enumeration: Bijections, Generating Functions, Binomial and Multinomial coefficients, Inclusion-Exclusion, Stirling’s Formula, Recurrence Relations.
- Hypergraphs: Sperner’s Theorem, LYM inequality, Erdos-Ko-Rado, Kruskal-Katona, Fisher’s Inequality, Baranyai, Ray-Chaudhuri-Wilson, Frankl-Wilson and Harper
- Partially Ordered Sets: Dilworth’s Theorem, linear extensions of posets, Distributive Lattices, Geometric Lattices, Birkhoff Representation Theorem, Dowling-Wilson, Möbius inversion, Weisner.
- Correlation Inequalities: Harris- Kleitman, FKG Inequality, Ahlswede-Daykin, Shepp (XYZ), BK inequality
- Ramsey Theory: Ramsey, infinite Ramsey, König tree lemma, probabilistic lower bounds, van der Waerden, statement of Sze-smerédi, Hales-Jewett theorem, Rado’s theorem (Partition Regularity and columns condition), Hindman’s theorem

1.2. Probabilistic Methods.
- Basics: Linearity of expectation, union bound, Bonferroni inequalities, Chebychev’s inequality, Markov’s inequality, Chernoff bounds, alteration methods.
- Second Moment Method: Application to threshold function for containing a fixed subgraph
- Local Lemma: Symmetric and general versions, Ramsey lower bounds, hypergraph coloring
1.3. Graph Theory.
- **Poisson Paradigm**: Janson’s inequality and the number of triangles in $G_{n,p}$, Brun’s sieve and the number of isolated vertices in $G_{n,p}$
- **Martingales**: Vertex and edge exposures, Azuma’s inequality and application to chromatic number, Talagrand’s inequality
- **Random Graphs**: $G_{n,p}$ versus $G_{n,M}$, monotone properties, Bollobás-Thomason existence of thresholds
- **Entropy**: Basic properties, Shearer’s lemma, Brégman’s Theorem

2. Partitions and Symmetric Functions

2.1. Partitions.
- **Definitions**: Definition of partitions, Ferrers diagrams, conjugates of partitions, skew diagrams
- **Tableaux**: Young tableaux, skew tableaux, Hook length formula

2.2. Symmetric Functions.
- **Algebra**: definition of associative algebra of symmetric functions, specializations, the $e/h/p/m$ symmetric functions, the $E/H/P$ generating functions and associated identities, involution $\omega$, inner product
- **Schur functions**: Definition of Schur functions, Kostka numbers, Littlewood Richardson rule
- **Bijections**: RSK, Cauchy Identity (and skew version), Jacobi Trudi, Gessel-Viennot