Oral Exam Syllabus

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I Experimental Math

Basics
Dynamic programming, Identities (Trig, Symmetric function), Proof machines, Proof by differentiation.

Hypergeometric Algorithms
Hypergeometric series: definition and canonical form. $\beta$ and $\Gamma$ functions. Similar hypergeometric terms.
  Identities: Gauss, Kummer, Saalschütz, Dixon, Clausen, Dougall
Algorithms: Sister Celine, Gosper, Zeilberger, Operator Algebra
  The Fundamental Theorem
Reference: Ptekovsek, Wilf, and Zeilberger, $A = B$.

Recurrences
Linear: c-Finite, p-Finite, (Non)-Homogenous, Telescoping, Solving.
Non-linear: Catalan Numbers.
II  Combinatorics and Graph Theory

II.1  Combinatorics

Enumeration
Bijections, partitioning, binomial/multinomial Coefficients, stirling numbers, inclusion/exclusion, generating functions

Posets and lattices
Hasse diagrams, chains and antichains, Dilworth’s theorem

Hypergraphs
LYM inequality, Sperner’s theorem, Erdös-Ko-Rado theorem, Van der Waerden’s theorem, Schur’s theorem, Fisher’s inequality, Ray-Chaudhuri, Wilson/Frankl Wilson, statement of Kruskal-Katona

Probabilistic Method
Linearity of expectation, Markov’s inequality, Chebyshev’s inequality, Chernoff-Hoeffding, method of alterations, 2nd moment method, Lovasz local lemma
II.2 Graph Theory

Basics
Basic graph definitions, trees, bipartite graphs, cycles and paths

Planarity
Euler’s formula, Kuratowski’s theorem, Wagner’s theorem, proofs that $K_5$ and $K_{3,3}$ are nonplanar

Matchings
Konig’s theorem, Hall’s theorem, Tutte’s theorem

Graph Algorithms
Max Flow/Min Cut theorem, Kruskal’s algorithm, Dijkstra’s algorithm

Colorings
Vertex coloring (Brook’s Theorem), edge coloring (Vizing’s Theorem), statement of weak/strong perfect graph theorem.

Extremal
Ramsey’s Theorem, bounds, Infinite Ramsey, Turán’s theorem. Erdős-Stone theorem