Syllabus for Oral Exam

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1 Major Topic: Partial Differential Equation

1.1 Classical solution of laplace equation

(1) Fundamental solution
(2) Mean value property
(3) Maximum principle
(4) Green’s representation formula
(5) Harnack’s inequality
(6) Liouville theorem
(7) Perron’s method
(8) Hopf lemma
(9) Schauder estimate
(10) Existence results by the method of continuity and fredholm alternative

1.2 Classical solution of heat equation

(1) Fundamental Solution
(2) Maximum Principles on bounded $\Omega_T$ and $R^n \times [0,T]$.
(3) Backward uniqueness by energy methods.

1.3 Wave equation

(1) d’ Alembert’s formula, Kirchohoff’s formula.
(2) Uniqueness and domain of dependence by energy methods.

1.4 Sobolev space

(1) Definition
(2) Density
(3) Extensions and Traces
(4) Rellich-Kondrachov compact imbedding theorem
(5) Poincare’s inequality and Morrey’s inequality
(6) $W_0^{1,n}$ case
1.5 Weak solution of second order Elliptic equation

(1) Lax-Milgram theorem and Fredholm alternative
(2) Maximum principle
(3) Moser Iteration method and Harnack’s inequality
(4) $L^p$ estimate and Interpolation theorem

1.6 Weak solution of second order Parabolic equation

(1) Definition of weak solution
(2) Energy estimates, Galerkin approximation
(3) Existence and uniqueness.
(4) Maximum principle.

2 Minor Topic: Riemannian Geometry

(1) Riemannian metric
(2) Levi-civita connection
(3) Curvature tensor, sectional curvature, ricci curvature, scalar curvature
(4) Geodesics and exponential map
(5) Jacobi fields, conjugate points
(6) First variation and second variation of arc length
(7) Bonnet-Myers Thm, Hopf-Rinow Thm, Cartan-Hadamard Thm
(8) Space form
(9) Hypersurface, second fundamental form

References

