ORAL QUALIFYING EXAM SYLLABUS

Vernon Chan

Algebraic Geometry

1. Sheaves: Presheaves and associated sheaves, morphisms of sheaves, stalks, pull-back and push-forward

2. Schemes: Spec and Proj, morphisms of schemes, generic points

3. First properties of schemes: Reduced, integral, noetherian schemes, morphisms of finite type, finite morphisms, open and closed immersions, fibre products

4. Separated and proper morphisms: Definition of separated and proper morphisms, valuative criteria, projective morphisms, scheme-theoretic images

5. Coherent and quasi-coherent sheaves: Definition of $\mathcal{O}_X$-modules, quasi-coherent and coherent sheaves, constructions of $\mathcal{O}_X$-modules, invertible sheaves, twisted sheaves, very ample invertible sheaves, vector bundles

6. Divisors: Weil divisors, Cartier divisors, Weil and Cartier divisor class groups, equivalence for locally factorial schemes, Picard groups

7. Projective morphisms: Criteria and characterization of projective morphisms, ample and very ample line bundles, blowing ups

8. Differentials: Derivations, sheaves of relative differentials, connection to nonsingularity, Bertini’s theorem, tangent sheaves, canonical sheaves, geometric genus

Cohomology Theory of Sheaves

1. Derived functors: Abelian categories, complexes, derived functors, $\delta$-functors

2. Cohomology of sheaves: The category of sheaves of $\mathcal{O}_X$-modules has enough injectives, Grothendieck’s vanishing theorem
3. **Cohomology of Noetherian Affine Schemes**: Cohomological characterization of noetherian affine schemes

4. **Čech Cohomology**: Definition, computation of sheaf cohomology groups for a noetherian separated scheme

5. **Cohomology of Projective Spaces**: Properties of cohomology groups of projective schemes, criterion for ampleness

6. **Extension Groups and Sheaves**

7. **Serre Duality Theorem**: Duality for $P^n_k$, dualizing sheaves, duality for projective schemes

8. **Spectral Sequences**

**Toric Varieties**

1. **Definitions**: Convex polyhedral cones, geometry of convex sets, affine toric varieties, fans and toric varieties, toric varieties from polytopes

2. **Singularities and compactness**: Local properties of toric varieties, surfaces, quotient singularities, one-parameter subgroups, limit points, compactness and properness, nonsingular surfaces, resolution of singularities

3. **Orbits, topology, and line bundles**: Orbits, fundamental groups and Euler characteristics, divisors, line bundles, cohomology of line bundles, canonical class

4. **Cohomology of smooth toric varieties**: Stanley-Reisner relation, self-intersection of invariant divisors

**References**


