Interfaces in Hierarchical (and finite D) Spin Glasses

Silvio Franz, LPTMS Orsay

Work in collaboration with:
M. Mezard, G. Parisi, T. Jorg

- Various definitions of interfaces in SG
- Overlap interfaces
- Dyson Hierarchical models
- 3d SG

Interfaces

Regions separating Thermodynamically Stable Phases of matter in coexistence conditions.

Order parameter variation in space.

Stable phases imply an interface free-energy cost growing with L (system size).

ΔF becomes finite at the lower critical D D_{LC}

e.g. :

Discrete O.P. values $\Delta F \sim L^{D-1}$

Continuous O.P. values $\Delta F \sim L^{D-2}$

Generating interfaces:

Fix the order parameter at different values at the boundaries along one direction.

Spin Glasses

Order parameter does not allow to distinguish different phases.

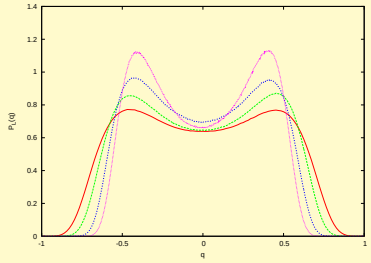
Different proposals on how to define interfaces

- Periodic-Antiperiodic B.C. (Bray-Moore) $D_{LC} \approx 2.5$.
- Zero modes associated to "gauge invariance" of Parisi solution (Brézin-De Dominicis)
- Overlap interfaces (F.,Parisi, Virasoro) $D_{LC} = 2.5$.

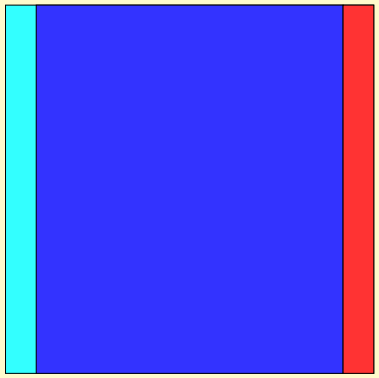
Astonishing prediction of RSB:

$Q(\sigma,\tau) = \frac{1}{N}\sum_{i=1}^{N}\sigma_i\tau_i$

$p(q)$ overlap PDF: Large at low temperature



yet: given two configurations the overlap is homogeneous in space.

$q(0) = q_1$  $q(L) = q_2$

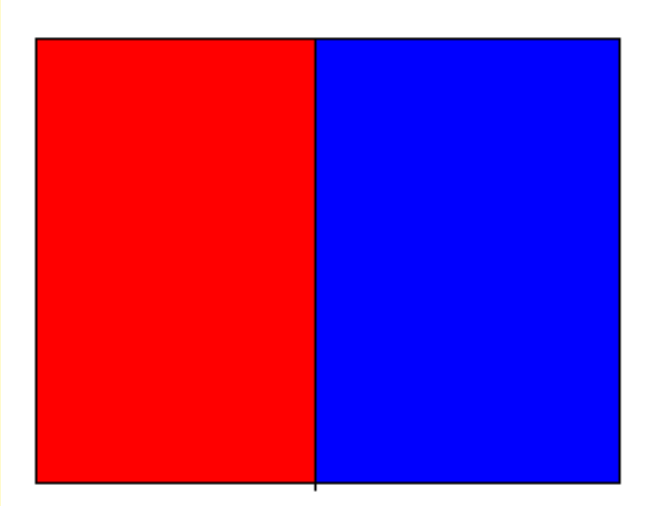
$$\Delta F_L(q_1,q_2) \propto L^{\theta_q}|q_1-q_2|^{b_q}$$

$$\theta_q = D - 5/2$$

$$b_q = 5/2$$

Equivalent method:

Fix the bulk value of the overlap in contiguous half spaces

$q(\Lambda_-) = q_1$  $q(\Lambda_+) = q_2$

For $q_1 \neq q_2$

$$P_L(q_1, q_2) \approx \exp\left(-\Delta F_L(q_1, q_2)\right)$$

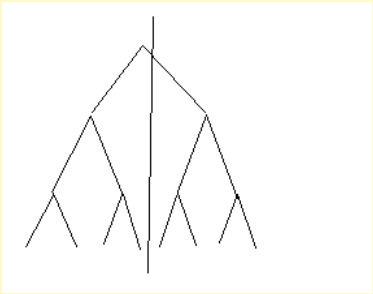
Easily measurable in numerical simulations.

$$\Delta F_L(q_1, q_2) \propto L^{\theta_q} |q_1 - q_2|^{b_q}$$

$$\theta_q = D - 5/2$$

$$b_q = 5/2$$

Hierarchical spin glasses



$$H^{(J)}_{k+1}(S_1,...,S_{2^{k+1}}) = H^{(J_1)}_k(S_1,...,S_{2^k}) + H^{(J_2)}_k(S_{2^k+1},...,S_{2^{k+1}}) + \frac{1}{2^{\sigma(k+1)}} \sum_{i<j}^{1,2^{k+1}} J_{ij} S_i S_j$$

J_{ij} i.i.d. Gaussian $N(0,1)$ ($E(J_{ij}) = 0 \quad E(J^2_{ij}) = 1$). Model defined for $\sigma \geq 1/2$ ($\sigma = 1/2$ SK limit).

$1/2 \leq \sigma \leq 2/3$ MF like transition

$2/3 \leq \sigma \leq 1$ Non-MF finite T transition

Combine non-classical critical point with RSB / Accessible to analytic treatment

Recursion

$Z_k[Q]$ Average partition function for n replicas with mutual overlaps Q_{ab}

$$Z_{k+1}[Q] = e^{-\beta^2 2^{(2-2\sigma)(k+1)} \sum_{a,b}^{1,n} Q_{ab}^2} \int dP Z_k[Q+P] Z_k[Q-P]$$

Interface exponent: $\theta_q = 2 - 2\sigma$

Prediction for b_q making a stability analysis supposing RSB

$q = (q_1 + q_2)/2, \quad u = q_1 - q_2$

$p(q)$ overlap PDF

$\Delta F_k(q_1, q_2) = 2^{2(1-\sigma)k} \beta^2 p(q) |u|^3$

$$P_k(u|q) \sim \exp\left(-2^{2(1-\sigma)k} \beta^2 p(q) |u|^3\right)$$

$b_q = 3$

$$P_k(u|q) \sim \exp\left(-2^{2(1-\sigma)k}\beta^2 p(q)|u|^3\right)$$

$b_q = 3$

What about the *unconditioned* $P_k(u)$?

Large deviation result: Behavior at finite u dominated by q^* s.t. $p(q^*) = max$

Suppose smooth crossover: Scaling variable $|u|^3\ 2^{2(1-\sigma)k}$

$$P_k(u) = 2^{\frac{2}{3}(1-\sigma)k} g(2^{\frac{2}{3}(1-\sigma)k} u)$$

$\theta_q/b_q = \frac{2}{3}(1-\sigma)$

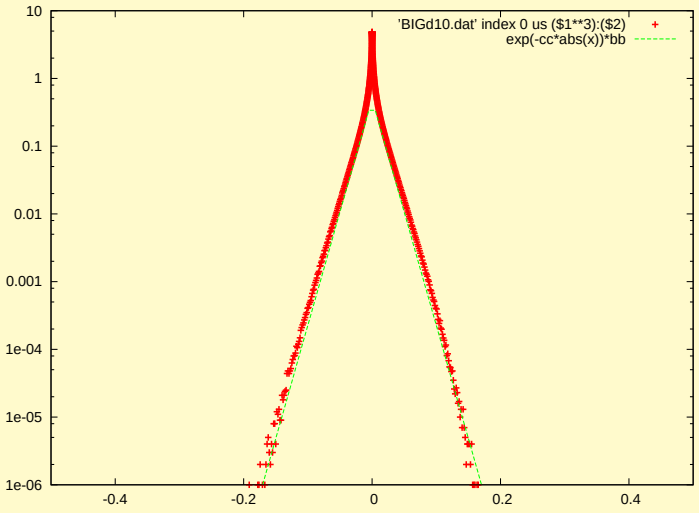
Results of simulations (I):

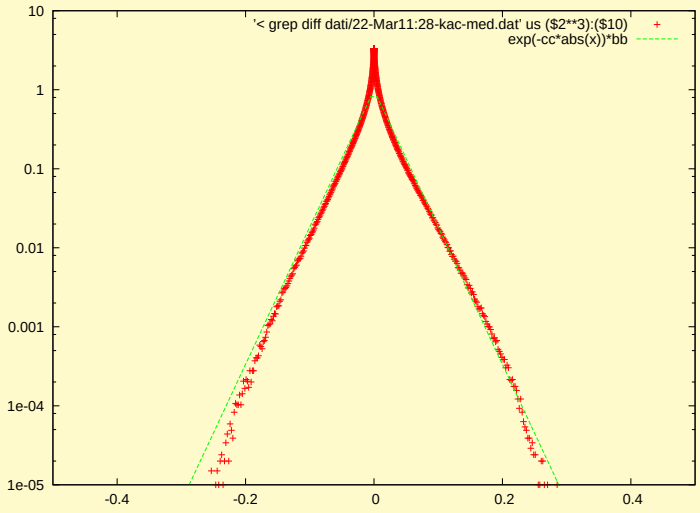
Diluted model in the same universality class. (Details upon demand...)

$\sigma = 0.567$ (MF region) $\qquad$ $\sigma = 0.707$ (Non-Classical region)

$T = 0.794 = 0.635 T_c$ $\qquad$ $T = 0.740 = 0.65\ T_c$

$L = 2^{10}$



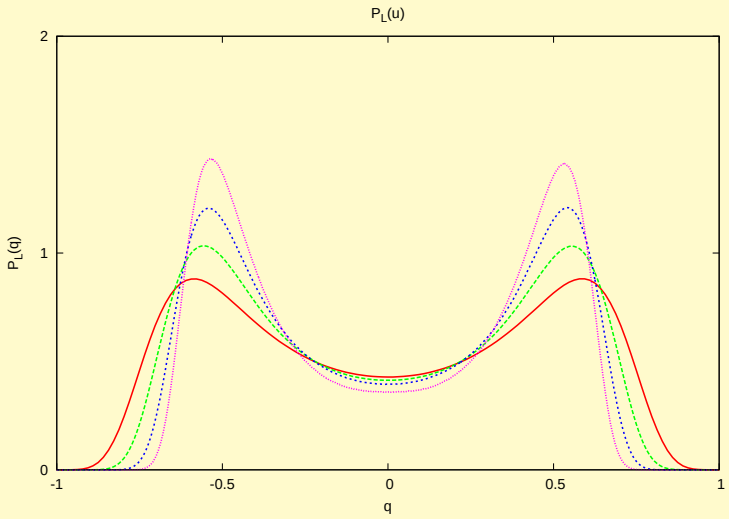


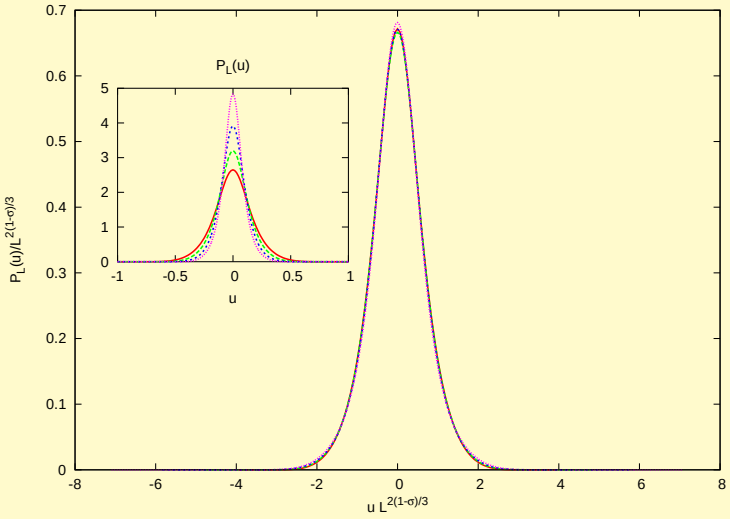
$\log P_k(u)$ versus u^3

Results of simulations (II):

$\sigma = 0.567$ (MF region)

$T_c = 1.25$





$L = 2^k \qquad k = 7, 8, 9, 10$

$T = 0.794 = 0.635 T_c$ $\theta_q / b_q = 0.28$

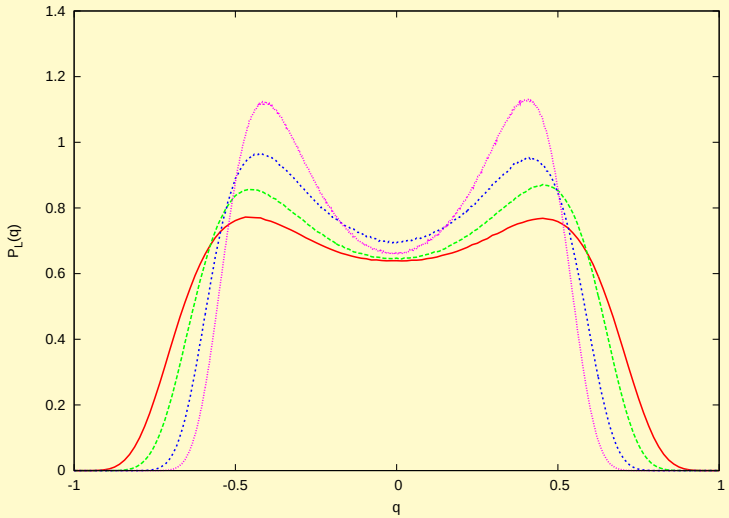
Notice the non-Gaussian shape of $P(u)$.

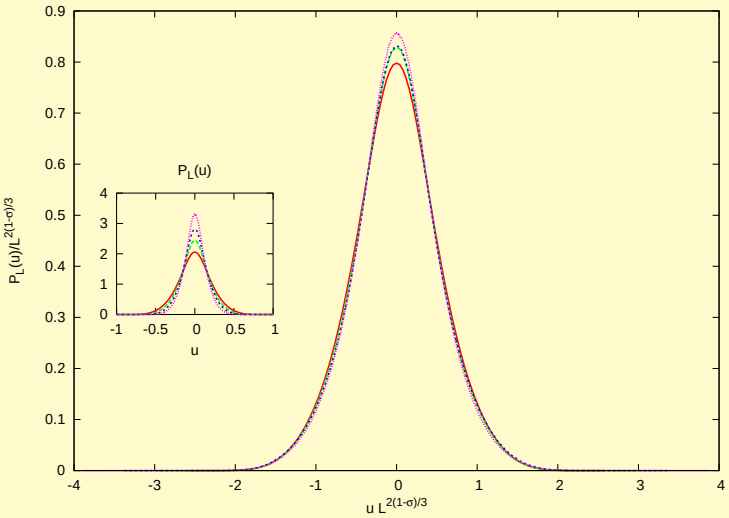
Results of simulations (III):

Diluted model in the same universality class.

$\sigma = 0.707$ (Non-classical region)

$T_c = 1.14$





$k = 7, 8, 9, 10$

$T = 0.740 = 0.65\ T_c$ $(\theta_q/b_q)_{theo} = 0.19$

$\theta_q/b_q = 0.22$ fits better the data

Results of simulations (IV):

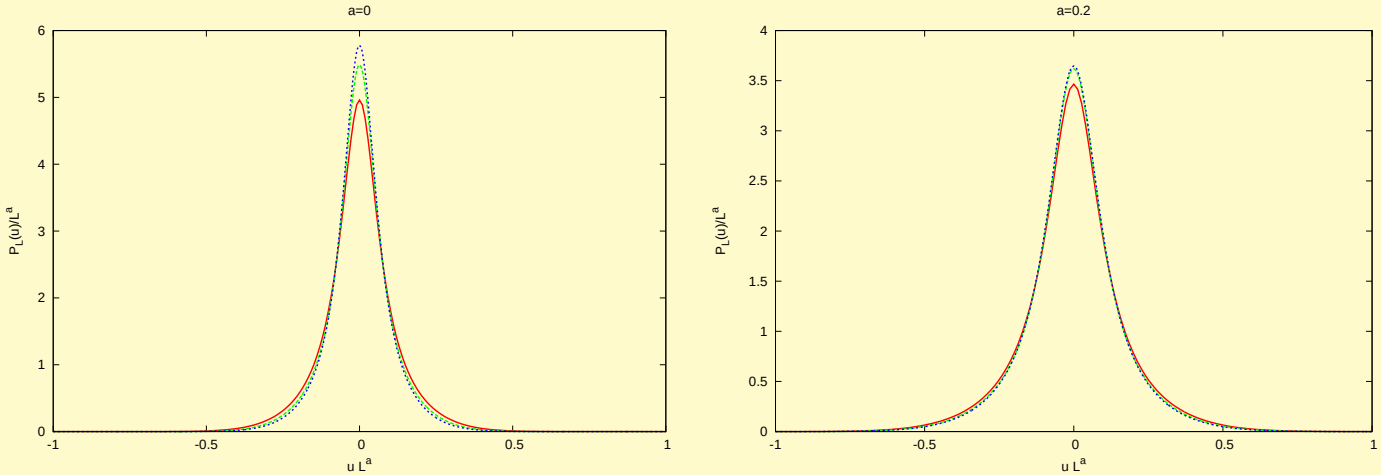
3D model in the universality class of Edwards-Anderson $T_c = 1.47$

$T = 1 = 0.68\, T_C$

MF RSB prediction: Scaling variable $L^{D-5/2} u^{5/2} \quad D = 3$

$L = 6, 8, 10$

$$P_L(u) = L^{1/5} g(uL^{1/5})$$



Again $\theta_q/b_q = 0.27$ fits better the data

Summary

- New method to study numerically overlap interfaces $P_L(q_1, q_2)$.
- RSB give predictions for the interface exponents θ_q and b_q.
- Good results in hierarchical and 3D spin glasses.
- Systematic numerical studies needed.
- Are MF exponents exact ?
- Hierarchical models $P_L(q|u)$ should lead to an independent measure of $p(q)$.