Periodic Ground States in Spin Systems with Long Range Competing Interactions

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joint work with

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1. **Pattern formation in thin films**

    - Introduction
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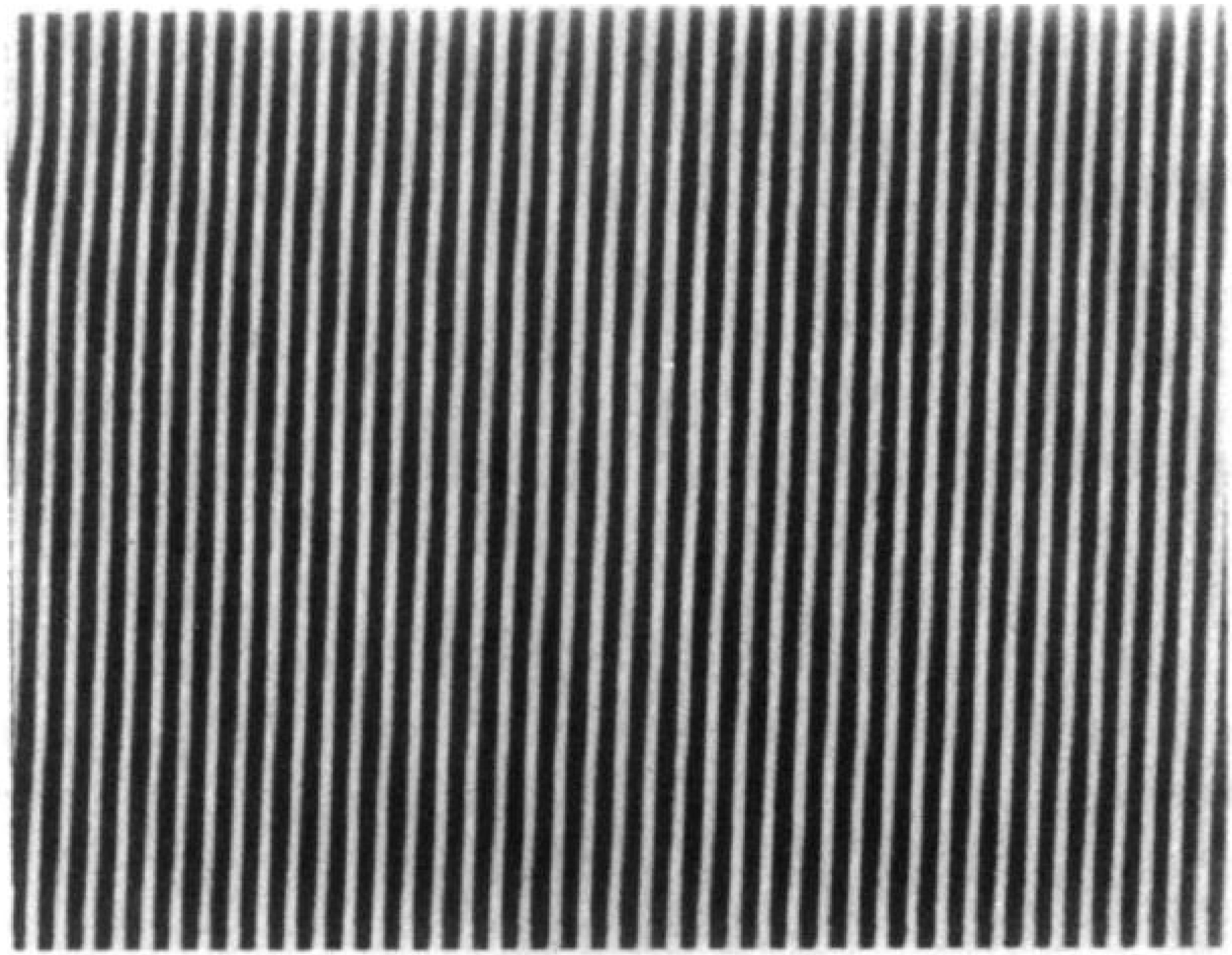
Introduction. Spontaneous formation of **striped** mesoscopic patterns is a "universal" phenomenon exhibited by a variety of quasi–2D systems, including:

thin magnetic films, polymer films, liquid-crystals, 2D electron gases, high-T_c superconductors, etc.

It is believed to be due to the **competition** between **short ranged ferromagnetic** and **long ranged dipole-dipole** (or **Coulomb**) interactions.



Ferrimagnetic garnet film on GGG

$H = 0$ and $T = 0.6T_c$, with $T_c = 192^o$ C

[M. Seul and R. Wolfe, PRA (1992)]

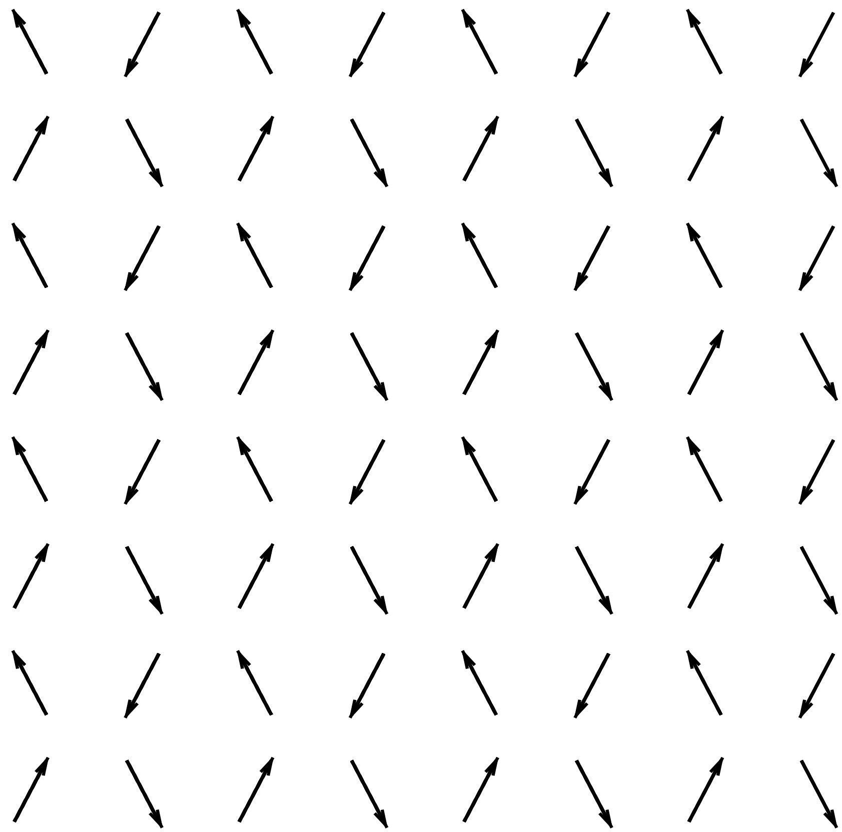
Simplest class of **spin models** for **thin magnetic films**

$$H \;=\; \sum_{\mathbf{x}\neq\mathbf{y}} \vec{S}_{\mathbf{x}}\,\hat{W}^{dip}(\mathbf{x}-\mathbf{y})\vec{S}_{\mathbf{y}} - J \sum_{<\mathbf{x},\mathbf{y}>} \vec{S}_{\mathbf{x}}\cdot\vec{S}_{\mathbf{y}} + \;\cdots$$

where $\vec{S}$ are classical Heisenberg spins and

$$W^{dip}_{ij}(\mathbf{x}) = -\partial_i\partial_j\frac{1}{|\mathbf{x}|} = \frac{1}{|\mathbf{x}|^3}\left(\delta_{ij} - 3\frac{x_i x_j}{|\mathbf{x}|^2}\right)$$

Known results. If $J = 0$ the ground state can be determined exactly, as proved by [Fröhlich-Israel-Lieb-Simon] and [Fröhlich-Spencer]. It displays in-plane uniaxial AF order and is continuously degenerate.



A ground state of the pure dipole system

The result are based on *reflection positivity* (RP).

Unfortunately RP is lost for any $J > 0$.

What happens for finite (and possibly large) J?

For large J, in the presence of anisotropy terms favoring alignment along coordinate directions (or perpendicularly to the plane), the best known variational state is a **periodic striped** state, whose stripes' size increases with J.

Main results. We consider the Hamiltonian

$$H_\lambda = \sum_{\mathbf{x}\neq\mathbf{y}} \vec{S}_{\mathbf{x}}\, \hat{W}^{dip}(\mathbf{x}-\mathbf{y})\vec{S}_{\mathbf{y}} - \sum_{<\mathbf{x},\mathbf{y}>} \left[J\vec{S}_{\mathbf{x}} \cdot \vec{S}_{\mathbf{y}} + \lambda(\vec{S}_{\mathbf{x}} \cdot \vec{S}_{\mathbf{y}})^2 \right]$$

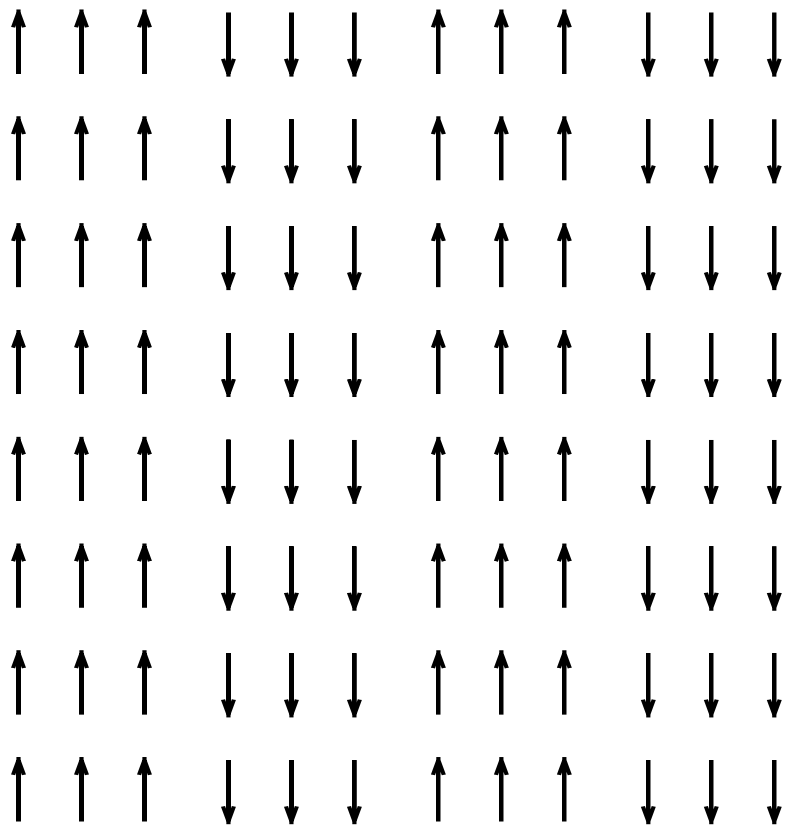
where $\lambda \geq 0$ and $\vec{S}_{\mathbf{x}}$ are in-plane spins, whose allowed directions are only $\uparrow, \downarrow, \rightarrow$ and $\leftarrow$.

The λ term has the effect of encouraging (anti)alignment, but this term alone cannot create periodic order.

Theorem. *For $J \geq 0$ and λ large enough, the specific ground state energy of H_λ in the thermodynamic limit is given by:*

$$\lim_{|\Lambda|\to\infty} \frac{1}{|\Lambda|} E_0(\Lambda) = \min_{h\in\mathbb{Z}^+} e(h)$$

where $e(h)$ is the specific energy of a striped configuration of period $2h$. On a torus of side divisible by the optimal period, the only ground states are the optimal periodic striped configurations.



A periodic striped ground state

Remarks.

(1) As far as we know, this is the only spin model with real dipole interactions, for which existence of periodic striped order has been proved.

(2) The condition on λ is not uniform in J. It is unclear whether the same result should be valid for large λ, uniformly in J, or even up to $\lambda = 0$.

Sketch of the proof.

For $\lambda = +\infty$ the dipoles are all (say) vertical.

Each column prefers to be aligned, either up or down: this is proved using RP.

The interacting columns are equivalent to a 1D spin system.

The energy $E(h_1, \ldots, h_M)$ of a sequence of stripes of sizes $h_1, \ldots, h_M$ can be bounded from below as:

$$E(h_1, \ldots, h_M) \geq \sum_{i=1}^{M} h_i \, e(h_i) \geq \Big(\sum_{i=1}^{M} h_i\Big) \min_h e(h)$$

The extension to finite λ is based on a Peierls' argument.

It is necessary to take into account screening and λ must be larger than const. $h^*(J)$.

Remarks.

(1) Technically, the *chessboard estimate* we use is obtained in the presence of open boundary condition. This new derivation tremendously simplifies the discussion related to the error terms produced by p.b.c.

(2) The same methods have been recently used to prove the existence of periodic minimizers for a class of 1D local mean field theories with long-range interactions.

References.

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3. A. Giuliani, J. L. Lebowitz, E. H. Lieb: *Periodic minimizers in 1D local mean field theory*, arXiv:0712.2330, to appear in Comm. Math. Phys.