Inference of Synaptic Weights in the Integrate-and-Fire Neuron Model

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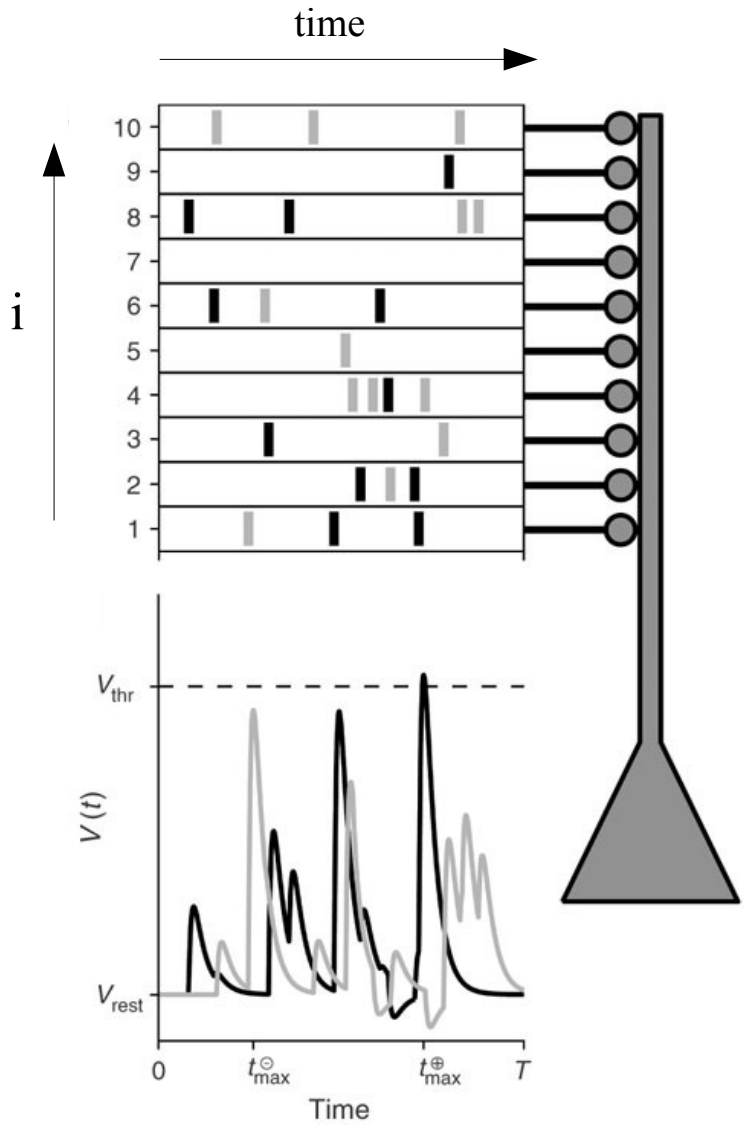
Rutgers, May 2008

A « Realistic » Neuron: the Integrate-&-Fire model

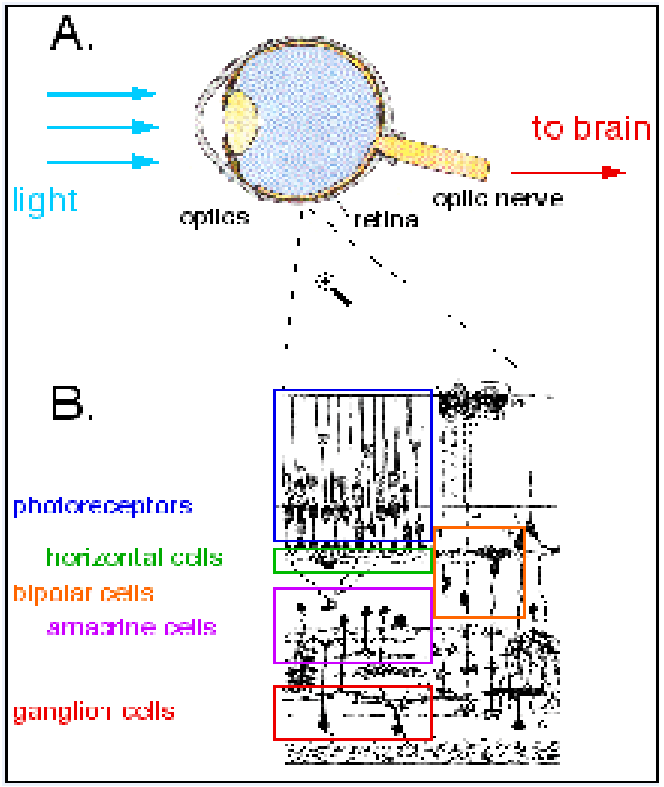


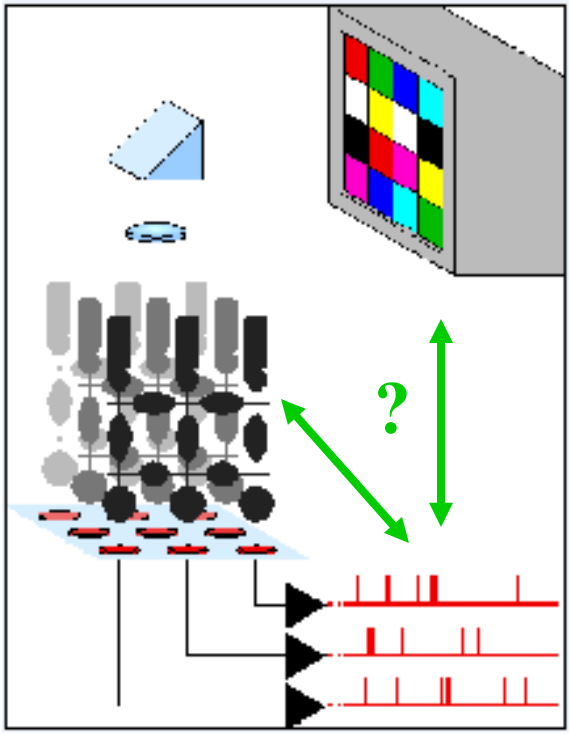
$$\frac{dV}{dt}(t) = -\,g\,V(t) + \sum_i J_i \sum_n C(t-t_{i,n}) + \eta(t)$$

Time-scale << 1/g

active or **silent**

Multi-electrode recording from retina





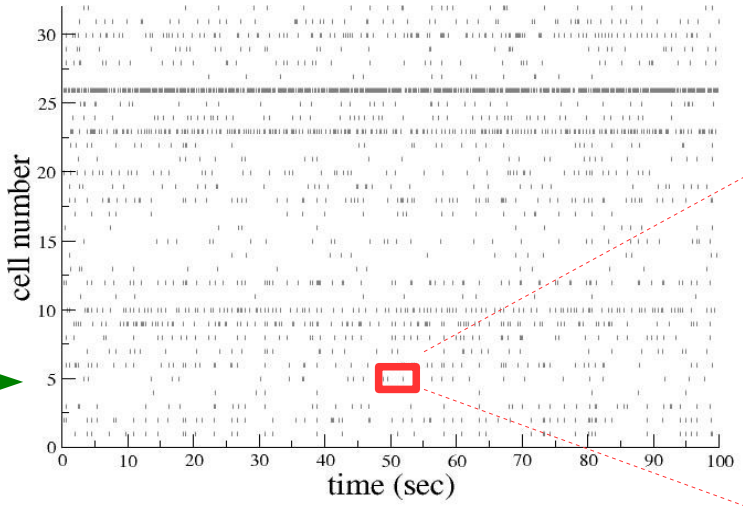
Berry et al. (2006): 40 ganglion cells, natural movies

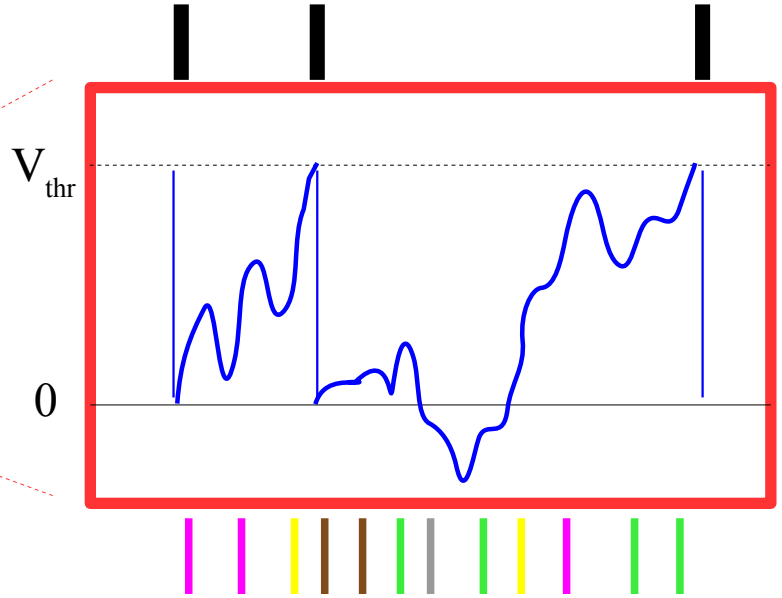
Meister et al. (1997): 60 ganglion cells in the dark, random flickering stimuli

Experiments typically run for 1 hour

Inference of Synaptic Weights from Raster Plots

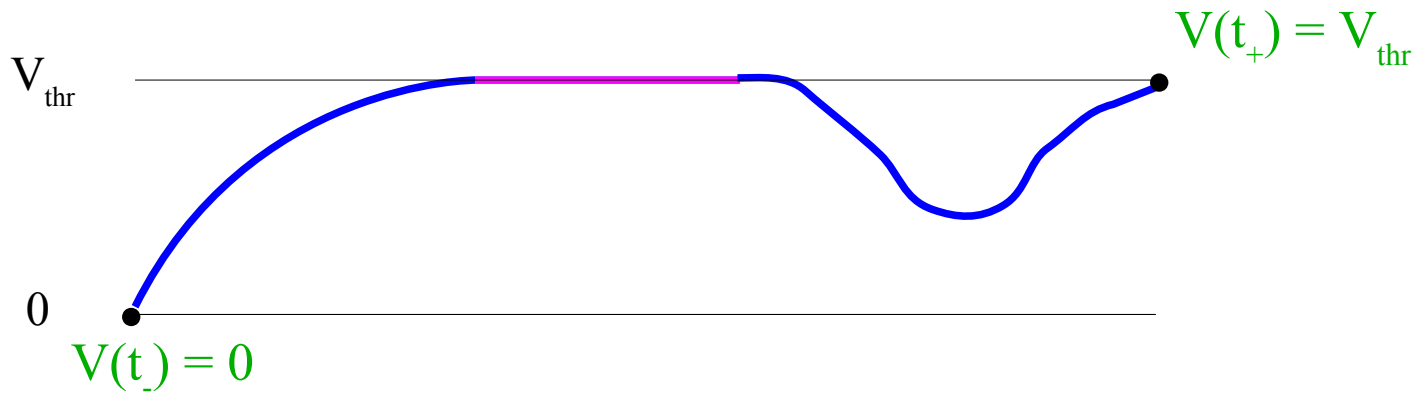
Couplings from cells i to j=5?





- $P[\{t_{i,n}\}|\{J_i\}]$ = product of FPT probabilities
- $\{J_i\}$ maximizing $P[\{J_i\}|\{t_{i,n}\}]$?
- Instanton calculation (variance of noise → 0):
  optimal path for V(t) conditioned by $\{t_{i,n}\}$

Optimal Paths for Potential and Noise



$$\left.\begin{aligned} \frac{dV}{dt}(t) &= -\,g\,V(t) + \sum_i J_i \sum_n C(t-t_{i,n}) + \eta(t) \\ \frac{d\eta}{dt}(t) &= +\,g\,\eta(t) \end{aligned}\right\} \quad or \quad \left\{\begin{aligned} & V(t)=V_{thr} \\ & \eta(t) = g\,V_{thr} - \sum_i J_i \sum_n C(t-t_{i,n}) \end{aligned}\right.$$

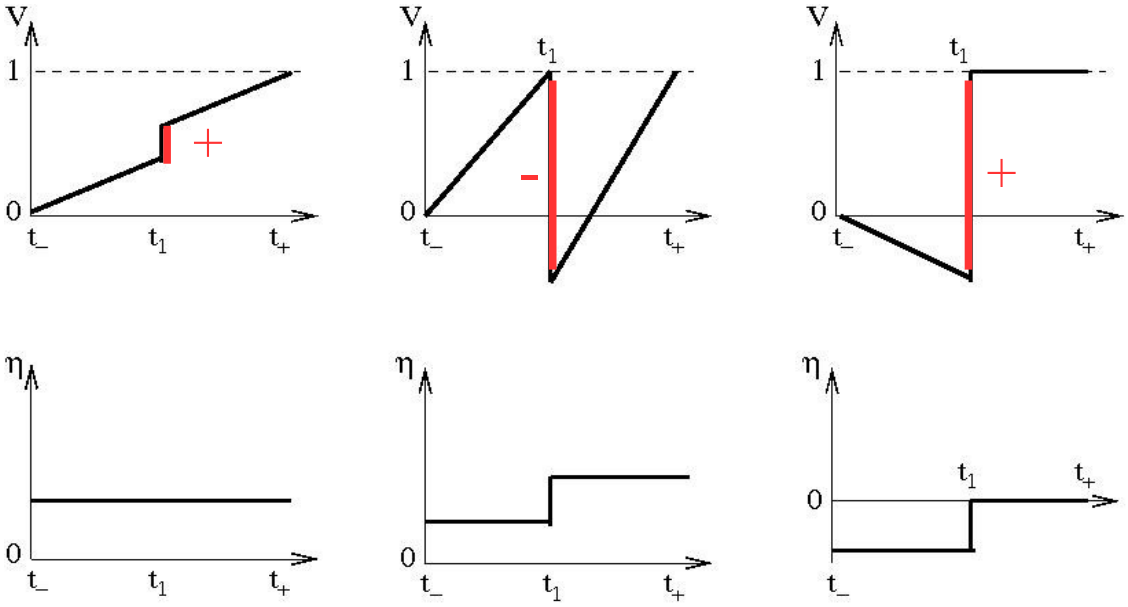
- Where are contact intervals? ... not easy to find out ...
- Limit-case of instantaneous synapses: $C(t)=\delta(t)$
- $g = 0$ (not essential): noise is constant, except over contact intervals

➔ Intervals reduce to isolated points coinciding with (some of the) entering spikes

➔ $\text{Noise}^{\text{after}}$ can take any value > $\text{Noise}^{\text{before}}$

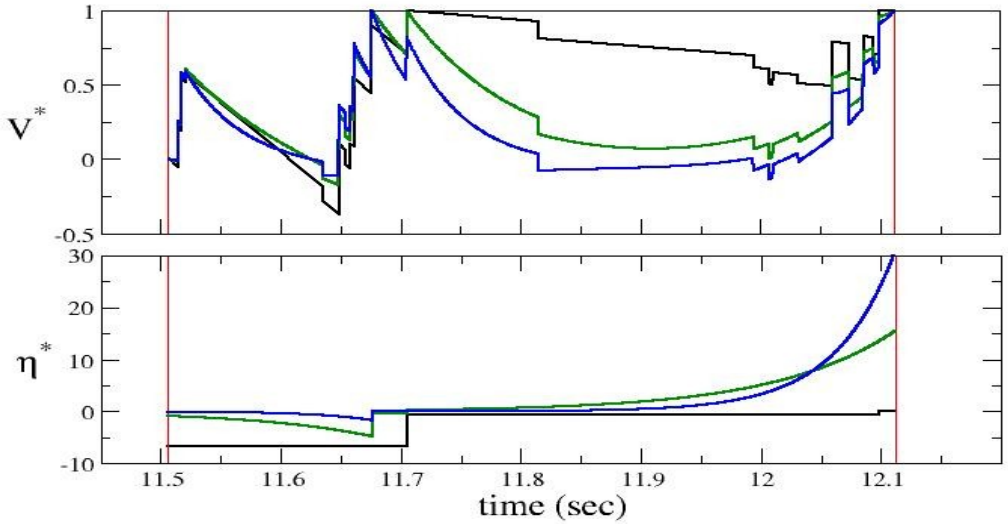
Looking for Contact Points

1 incoming spike



More incoming spikes

(20 spikes from 31 cells)

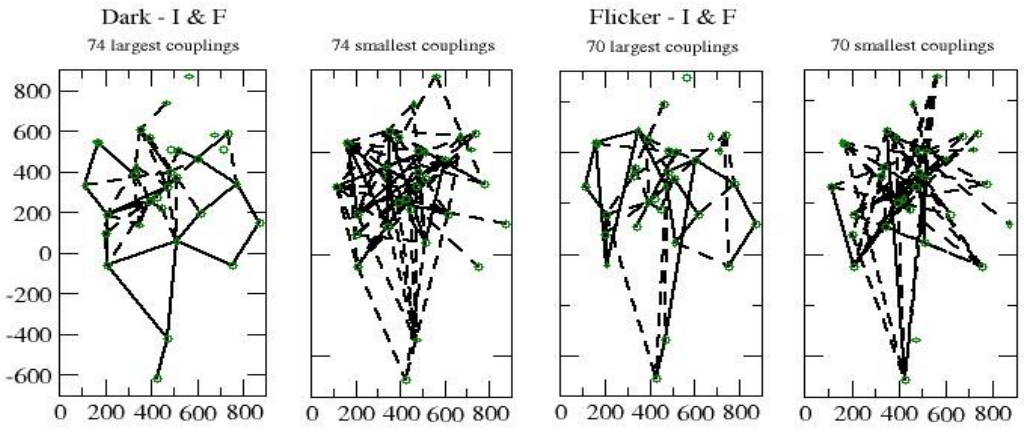


g=0, 10, 20 sec^{-1}

Searching for Optimal Interactions

- Optimal action (log-likelihood) is a convex function of $\{J_j\}$ → gradient descent
- Repeat for all cells → matrix of interactions $\{J_{ij}\}$
- Applications to multi-electrode recordings of retinal ganglion cell activity:
  (up to 60 cells so far → 1770 couplings)

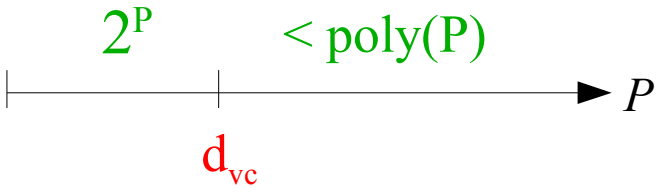
Allow us to study various aspects of interactions of biological interest:
spatial structure, symmetry, dependence on stimulus, on time-scales, ...



Computational Capabilities of the I&F neuron

Input activity $\xi_i(t) \rightarrow$ Output $\sigma = 0$ (silent) or 1(active)

Set of P inputs $\{\xi_i^\mu(t)\}$ for $\mu = 1,\ldots,P$: how many output sets $\{\sigma^\mu\}$ are realizable?



Vapnik
Chervonenkis
1971

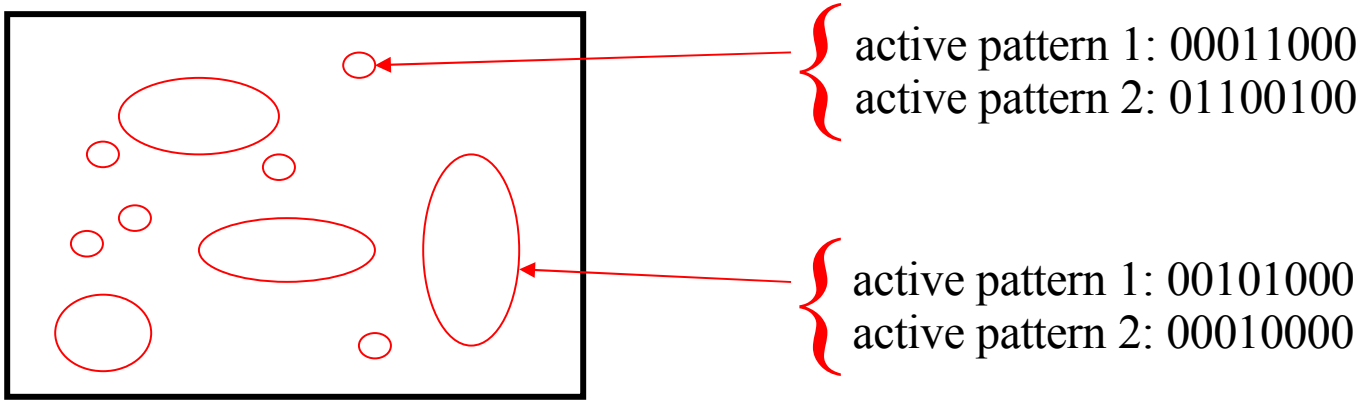
Dimensionless time: $K = T \times g$

- $K << 1$: only $\xi_i^\mu = \langle \xi_i^\mu(t) \rangle$ matters $\rightarrow$ perceptron

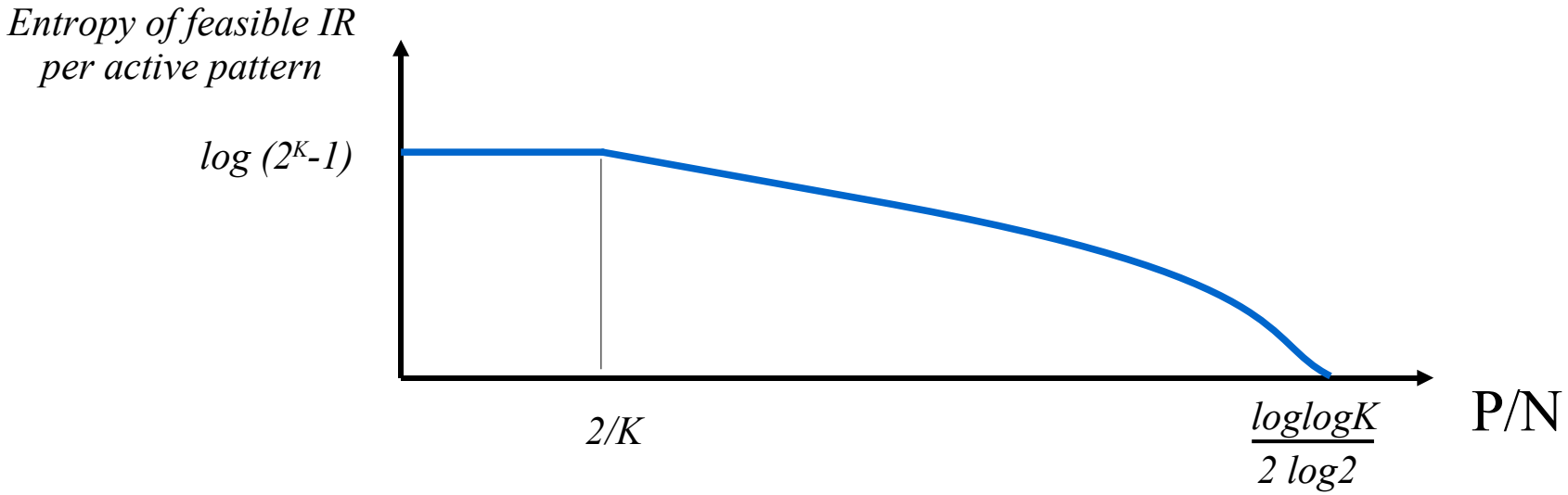
Gardner 1987
Gardner, Derrida 1988

- $K >> 1$? $1/\log K < d_{vc}/N < \log K$ …

I&F Neuron and Internal Representations



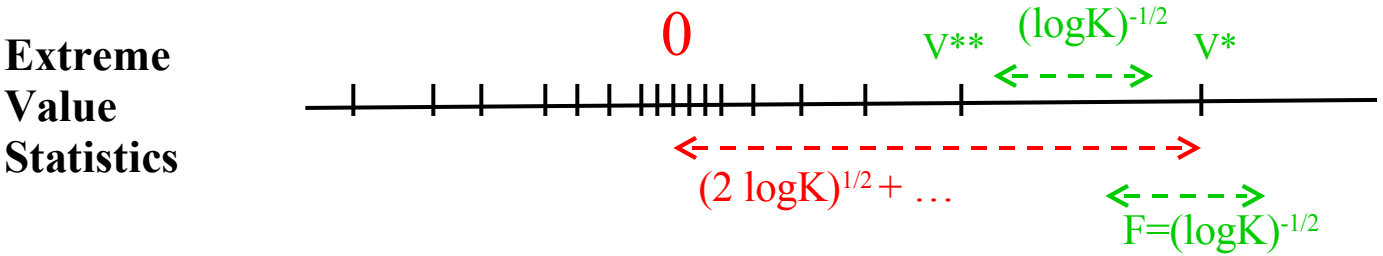
How many domains are not empty i.e. IR are feasible?



Robustness against Time and Space Correlations

$$\frac{d_{vc}}{N} = \frac{\text{loglogK}}{2\ \log 2}$$

obtained for uncorrelated patterns (in time, space) ....



very robust against correlations! (Berman, 1964)