From the Laplace equation to harmonic maps: the impact of topology

Topic

In this lecture I shall discuss two fairly well-known PDE's.

The first one,

$$\Delta v := \frac{\partial^2 v}{\partial x_1^2} + \frac{\partial^2 v}{\partial x_2^2} + \frac{\partial^2 v}{\partial x_3^2} = 0, \qquad v\colon \text{ball} \subset \mathbb{R}^3 \to \mathbb{R},$$

describes the equilibrium temperature of a uniform body. The properties of the function v have been studied since the 18^{th} century and they are completely understood by now.

A much more recent problem is the equation

$$-\Delta u = u|\nabla u|^2, \qquad u : \text{disk} \subset \mathbb{R}^2 \to S^1$$

which models the configuration of superfluids in physics. The solutions u to this problem are called harmonic maps.

Although these two equations have totally different structures, both arise as minima of the same energy functional

$$E(w) = \int |\nabla w|^2.$$

I will start my lecture with some historical background about the first problem. Then I will explain why topology makes the second equation much more difficult to solve and also how one can attempt to find solutions to it.

Lecturer

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