

SYLLABUS FOR WRITTEN QUALIFYING EXAMS

PREPARED BY THE AD HOC COMMITTEE ON THE QUALIFYING EXAM

Advanced Calculus and Elementary Point Set Topology

- Properties of the reals such as Bolzano-Weierstrass and Heine-Borel.
- Differential calculus of \mathbb{R}^m -valued functions on subsets of \mathbb{R}^n . Continuity and uniform continuity, differentiability, (in the sense of Fréchet), partial derivatives, Jacobians, implicit function theorems, functions of bounded variation,. Elementary differential equations.
- Infinite sequences and series of numbers and functions. Absolute and uniform convergence, equicontinuity.
- Riemann and Riemann-Stieltjes integration, fundamental theorem of calculus.
- Line integrals, surface integrals, flux and circulation of a vector field. The theorems of Stokes and Green and the divergence theorem. Change of variable in multiple integrals. Closed and exact differential, independence of path.
- Basic topological properties such as connectivity, compactness, separability and the separation axioms.
- Basic topological constructions such as product spaces, compactifications.
- Metric spaces and completeness.

Real Analysis

- Definition and elementary properties of Lebesgue measure.
- Measurable functions, simple functions.
- The Lebesgue integral and its elementary properties.
- Convergence theorems.
- Various types of convergence, such as almost everywhere, in measure, in mean.
- Multiple integrals and changing the order of integration (Fubini's Theorem).

Complex Analysis

- The complex derivative, functions holomorphic (i.e., analytic) on an open set, Cauchy-Riemann equations, harmonic functions, conformal maps, mapping properties of the exponential and logarithm.
- Complex integration. The Cauchy Integral Theorem and the Cauchy integral formula. The residue theorem and evaluation of definite integrals.
- Taylor and Laurent expansions of holomorphic functions. Behavior near isolated singularities.
- The Cauchy inequalities and the maximum principle.
- The Riemann sphere and the extended complex plane, fractional linear transformations (i.e., Möbius mappings). Statement of the Riemann mapping theorem.

Algebra

- Vector spaces and elementary concepts of linear transformations, matrices, bases, change of bases.
- Group theory, including basic concepts, normal subgroups, the isomorphism theorems, the alternating and symmetric groups, direct products of groups, groups acting on sets, the Sylow theorems.
- Bilinear and quadratic forms, including inner product spaces, alternating and symmetric forms.
- Principal ideal domains (PIDs) and unique factorization. The fundamental theorem of finitely generated modules over a PID and applications to Jordan canonical form.