EXAM SYLLABUS

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1. Algebraic Number Theory
   (1) Number Fields: Integral Bases, Different, Discriminant
   (2) Ramification, Splitting of Primes
   (3) Ideal Class Group, Minkowski Bound, Finiteness of the Class Number
   (4) Dirichlet’s Unit Theorem
   (5) Quadratic and Cyclotomic Fields

2. Algebraic Geometry
   (1) Sheaves, Schemes, Sheaves of Modules
   (2) Divisors, Riemann-Roch for Curves
   (3) Differentials, Genus
   (4) Cohomology of Sheaves, Cohomology of Noetherian Affine Schemes and projective
       space, Serre Duality

3. Linear Algebraic Groups
   (1) Correspondence Between Groups and Lie Algebras in Characteristic Zero
   (2) Jordan-Chevalley Decomposition, Diagonalizable Groups
   (3) Reductive and Semisimple Groups
   (4) Structure of Borel Subgroups
   (5) Weyl Group
   (6) Bruhat Decomposition
   (7) Parabolic Subgroups
   (8) Examples of Classical Groups
   (9) Root Systems and Dynkin Diagrams

4. Elliptic Curves
   (1) Elliptic Curves over \( \mathbb{C} \)
   (2) Elliptic Curves over finite fields, The Hasse Bound
   (3) Elliptic Curves over local fields
   (4) Elliptic Curves over Global fields, The Mordell-Weil Theorem