Oral Exam Syllabus

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Part I: Partial Differential Equations

1. Laplacian Equation
   - Fundamental solution.
   - Properties for harmonic functions: Mean-value formulas, Maximum principle, Regularities, Liouville's theorem, Harnack’s inequality.
   - Green’s function.
   - Variational method for the Dirichlet’s principle.

2. Heat Equation
   - Fundamental solution.
   - Mean-value formulas, Maximum principle, Regularities.
   - Backward Uniqueness.

3. Wave Equation
   - Solution for homogeneous and nonhomogenous equations (d’Alembert’s formula, Kirchhoff’s and Poisson’s formulas, etc.)
   - Energy methods.

4. Sobolev Space
   - Basic properties.
   - Approximations, Extensions, and Traces.
   - Gagliardo-Sobolev-Nirenberg inequality, Morrey’s inequality, Poincare’s inequality.
   - Compact imbedding.

5. Second-Order Elliptic Equations
   - Variational formulations and existence of weak solutions.
   - Regularities.
   - Maximum principle, Harnack’s inequality.
   - Eigenvalues and eigenfunctions.

Part II: Functional Analysis

1. Banach Space
   - Hahn-Banach Theorem, Separation of convex sets.
   - Conjugate convex functions.
   - Baire Category theorem, Uniform bounded principle, the open-mapping theorem and closed graph theorem.
   - Weak and Weak* topology, Reflexivity, Separability.

2. Hilbert Space
   - Projection onto a convex set.
   - Riesz representation Theorem.
• The theorems of Stampacchia and Lax-Milgram.
• Hilbert sums and orthonormal bases.

(3) Compact Operator
• Fredholm alternative.
• The spectrum of compact operators.
• Spectral decomposition of self-adjoint compact operators.

(4) Sobolev functions in real line.
• The properties of $W^{1,p}(I)$.
• Variational formulation and spectrum analysis for certain ODE problems.

REFERENCES