Problem Set 1, Math 350, Fall 2017

(1) Find the space of polynomials \( f(x) = \{ax^2 + bx + c\} \) satisfying \( f(-1) = -1, f'(-1) = -1 \).

(2) Show that the set of functions \( f : S \to \mathbb{R} \) from a set \( S \) to the real numbers \( \mathbb{R} \) with addition given by \((f + g)(x) = f(x) + g(x)\) and scalar multiplication given by \((cf)(x) = c(f(x))\) satisfies axiom (VS4).

(3) Show that the set of polynomials \( f(x) \) of a single real variable \( x \) with addition given by \((f + g)(x) = f(x) + g(x)\) and scalar multiplication \((cf)(x) = f(cx)\) is not a vector space.

(4) Show that if \( V \) is a vector space and \( v \in V \) and \( c \in F \) and \( cv = 0 \) then either \( c = 0 \) or \( v = 0 \).

(5) Prove that \( \{(1, 1, 0), (1, 1, 1), (0, 1, 1)\} \) is linearly independent over \( \mathbb{R} \) but linearly dependent over \( \mathbb{Z}_2 \).