Branching Theorems for Compact Symmetric Spaces

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A compact symmetric space, for purposes of this lecture, is a quotient G/K, where G is a compact connected Lie group and K is the identity component of the subgroup of fixed points of an involution. A branching theorem describes how an irreducible representation decomposes under restriction to subgroups. The lecture deals with branching results, partly proved and partly not, for the passage $G \to K_1 \times K_2$, where $G/(K_1 \times K_2)$ is any of the three quotients $U(m+n)/(U(m) \times U(n))$, $SO(m+n)/(SO(m) \times SO(n))$, or $Sp(m+n)/(Sp(m) \times Sp(n))$, with $m \geq n$. For each of these compact symmetric spaces, one associates another compact symmetric space G'/K_2 with the following property: To each irreducible representation (σ, V) of G whose space V^{K_1} of K_1-fixed vectors is nonzero, there corresponds a canonical irreducible representation (σ', V') of G' such that the representations $(\sigma|_{K_2}, V^{K_1})$ and (σ', V') are equivalent. For the situations under study, G'/K_2 is equal respectively to $(U(n) \times U(n))/\mathrm{diag}(U(n))$, $U(n)/SO(n)$, and $U(2n)/Sp(n)$, independently of m. Hints of the kind of "duality" that is suggested by this theorem/conjecture date back to a 1974 paper by S. Gelbart.