DIAGRAMS, SINGULARITIES, AND THE REPRESENTATION THEORY OF LOCAL RINGS

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I will discuss the recent classification of local rings of finite representation type. These are commutative local rings having only finitely many indecomposable representations (or modules) satisfying a certain reasonable condition. (The modules in question are called *maximal Cohen-Macaulay* modules. They are the natural generalization of the *lattices* one encounters in algebraic number theory.) A beautiful theorem proved by Buchweitz, Greuel, Knörrer and Schreyer in 1987 states that a local ring of the form $\mathbb{C}[[X_0, \ldots, X_d]]/(f)$ has finite representation type if and only if the singularity defined by the power series f has *finite deformation type* in the sense discussed by V. I . Arnold in his address to the 1974 International Congress.

I will describe recent results on ascent and descent of finite representation type that provide a general characterization of the *excellent* local rings of finite representation type. What makes the general case much more difficult is the failure of the Krull-Schmidt uniqueness theorem: A module may be expressed in many different ways as a direct sum of indecomposable modules.