TITLE: Some surprising facts about real analyticity, optimal control, and subanalyticity of Carnot-Carathéodory distances

ABSTRACT: a theorem on "weak regularity with choice" for optimal controls proved by the speaker has turned out to have surprising consequences in recent years. The theorem itself says, among other things, that for real-analytic optimal control problems that have a unique solution, the optimal control is real-analytic on an open dense subset of the interval on which it is defined. (If the problem has more than one solution, then the theorem says that there always exists one solution having this property.) This result is surprising and worth discussing because (a) it is a regularity theorem but it is not proved using estimates, (b) the natural C^∞ analogue is completely false, (c) the proof involves disproportionately strong tools (resolution of singularities), considering the weakness of the conclusion, (d) no better results seem to exist, suggesting that perhaps the conclusion is the best possible after all, although this is far from having been proved, (e) the result has found unsuspected applications in the study of other problems. In particular, it has played a crucial role in the proof by A. Agrachev and J.-P. Gauthier—completed in April 2000—of a far-reaching result on subanalyticity of Carnot-Carathéodory distances for generic real-analytic subriemannian structures involving a distribution of rank ≥ 3. The talk will discuss the weak regularity result, outlining its statement, background, proof, and subsequent history (all four of which are rather peculiar), as well as the context, significance, and proof of the Agrachev-Gauthier theorem, emphasizing in particular the role of the earlier result in the proof of the latter one.

The statement and proof of the weak regularity theorem can be found in the speakerś 1996 paper

Some optimal control applications of real analytic stratifications and desingularization

available as a Postscript file at

http://www.math.rutgers.edu/~sussmann/currentpapers.html