SYLLABUS

- Chapters 1, 2, 3, and 4.
- The proofs of the limit laws will not be required on the midterm.
- The proof of \( \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \) will not be required on the midterm. Instead, students are expected to know the statement and to be able to apply it. In particular, students are expected to be able to prove that \( \lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta} = 0 \) using \( \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \).

Questions similar to the following may be on the exam:

Determine whether the following limit exists. If it exists, compute it. Do not use l'Hôpital’s rule.

\[
\lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta}.
\]

- Students are expected to know the statement of Theorem 2 on p102 and to be able to prove it for particular cases of the rational function. Any computation of limits at infinity of a rational function should be accompanied by full justification; quoting Theorem 2 on p102 instead of a full justification will result in a score of zero. For instance, if the exercise in Example 2 on p102 is on the exam, then the solution in the textbook would receive full credit, while writing just “0 by a Theorem” would result in a score of zero.

Exercises similar to the following may be on the exam:

1. Calculate \( \lim_{x \to \infty} \frac{20x^2 - 3x}{3x^2 - 4x^2 + 5} \). Do not use l'Hôpital’s rule.
2. Calculate \( \lim_{x \to \infty} \frac{20x^2 - 3x}{3x^2 - 4x^2 + 5} \).

In the case of 2 above, any correct method may be used.

- Students are expected to know the first and second list of derivatives and the first list of indefinite integrals [see the “Handouts” on the website].

- Students are expected to be able to prove all formulas on the first and second list of derivatives with the exception of \( \frac{d}{dx} e^x \). [see for instance p165 and p179].

Note regarding the proof of the power rule: only the cases of integer exponents are required.

- The proofs of the following theorems are not required on the midterm: the chain rule, the theorem about the derivative of the inverse on p178, Fermat’s theorem, the Mean Value Theorem, l'Hôpital’s rule.

THEORETICAL QUESTIONS

The exam will likely contain at least one “theoretical” question. Such questions are meant to evaluate students’ conceptual understanding. Here are just a few examples of theoretical questions.

Example 1. Define continuity of a function at a point. Define continuity of a function on an interval. Give an example of a function defined on a closed interval \( I \) which is not continuous at one point of \( I \) and continuous everywhere else.

Example 2. Define differentiability of a function at a point. Define differentiability of a function on an open interval. Prove that if \( f \) is differentiable at \( c \), then \( f \) is continuous at \( c \). Does the converse hold?
Example 3. State the rigorous definition of a limit of a function at a point and then prove that 
\[ \lim_{{x \to 2}} x^2 = 4. \]

Example 4. Obtain with proof the derivative of \( x^n \) where \( n \) is an arbitrary integer.

Example 5. Prove Rolle’s theorem.

Example 6. Assume that \( f : [a, \infty) \to \mathbb{R} \) is continuous on \( [a, \infty) \), differentiable on \( (a, \infty) \), and \( f'(x) > 0 \) on \( (a, \infty) \). Prove that \( f \) is increasing on \( [a, \infty) \).

Example 7. Let \( f(x) = \tanh x \). Find the domain \( D \) of \( f \), differentiate \( f \), find a “maximal” subset of \( D \) on which \( f \) is one-to-one, find the range of \( f \) on this maximal subset, the domain of the corresponding inverse function \( f^{-1} \), the derivative of \( f^{-1} \).

REMINDER

Any statement and any graph should be fully justified, unless otherwise indicated within an exam question.

If a certain exam problem is purely computational [such as the computation of a derivative/indefinite integral], then all steps of the computation should be clearly presented.