Due at the beginning (first 20 minutes) of the workshop on Monday, September 10

Please write your full name in capital letters on the front page of your homework, staple your homework, and write neatly; assignments that fail to satisfy these conditions may be disregarded. Provide full justification for your statements. Solutions without justification will receive little or no credit.

Hand in:
1.1: 42 (2 points), 44 (2 points), 48 (2 points), 60 (3 points), 62 (1 point);
1.2: 40 (5 points), 48 (5 points);
1.3: 34 (5 points);
1.4: 52 (5 points);

One more Problem. (20 points) Consider the quadratic function
\[ f(x) \equiv ax^2 + bx + c, \]
where \( a, b, c \) are real numbers such that \( a \neq 0 \). Let \( D \equiv b^2 - 4ac \) be the discriminant of \( f \).

(i) Assume that \( D < 0 \). Prove that \( f \) has no real roots.

(ii) Assume that \( D = 0 \). Prove that \( f \) has exactly one real root; determine a formula for this root in terms of (some of) \( a, b, c \).

Prove that if \( a > 0 \), then
\[ f(x) \geq 0 \quad \text{for all real numbers } x. \]

Prove that if \( a < 0 \), then
\[ f(x) \leq 0 \quad \text{for all real numbers } x. \]

(iii) Assume that \( D > 0 \). Prove that \( f \) has exactly two real roots; determine a formula for these roots in terms of \( a, b, c \). Let \( x_1 \) and \( x_2 \) be the two roots of \( f \) and assume that \( x_1 < x_2 \).

Prove that if \( a > 0 \), then
\[ f(x) > 0 \quad \text{whenever } x \in (-\infty, x_1) \cup (x_2, \infty) \]

and
\[ f(x) < 0 \quad \text{whenever } x \in (x_1, x_2). \]

Prove that if \( a < 0 \), then
\[ f(x) < 0 \quad \text{whenever } x \in (-\infty, x_1) \cup (x_2, \infty) \]

and
\[ f(x) > 0 \quad \text{whenever } x \in (x_1, x_2). \]

N.B. By (iii) above, if \( D > 0 \), then \( f \) has the same sign as the leading coefficient \( a \) outside “the interval of the roots” and the opposite sign to the leading coefficient on “the interval of the roots”.

No justification needed for: 1.1 – 42, 44, 62.
1. Let \( d_1 \) be a line of slope \( m_1 \) and \( d_2 \) a line of slope \( m_2 \). Prove that \( d_1 \) and \( d_2 \) are perpendicular if and only if \( m_1 m_2 = -1 \).

It is your duty to make sure that you understand why points were taken off your homework and what the correct solution in each case is. You should therefore analyze your graded homeworks carefully and ask questions during the workshops and office hours.

---

1 Regarding the questions that you are not supposed to hand in:

In some cases (e.g. 1.1/5,6) there is no need to write down much or anything at all. Just read the question, think about it, and make sure that you know how to answer it (with justification if the case is).