FULL NAME IN CAPITAL LETTERS:

SIGNATURE:

ID #:

SECTION:

Instructions:
• Turn off and put away all mobile phones, computers, iPods, etc.
• Show answers and arguments in the space provided. You may use the back of the pages also, but indicate clearly any such material that you want marked. Answers given without supporting work or justification may receive zero credit.
• This is a closed book exam. No calculators. No formula sheets. No questions. No textbooks or additional materials.
• Hand in the exam no later than 3:00pm.
• Use blue or black ballpoint pens please. Answers written in pencil will be disregarded.

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<th>Question</th>
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1. Compute the following derivatives:

(a) (4 points) \([(2x^3 + 3x + 1) \cos x]^{\prime}\)

(b) (4 points) \(\left(\frac{\sin x}{x^2 + 2x + 5}\right)^{\prime}\)
1. (continued) Compute the following derivatives:

(c) (5 points) \( \left( \sqrt{\frac{x}{3x+1}} \right)' \)

(d) (5 points) \([e^{-2x+5}(3x + 1001)]'\)
2. (a) (5 points) Define continuity of a function at a point.

(b) (2 point) Define continuity of a function on $\mathbb{R}$. (In other words, assume that $g : \mathbb{R} \rightarrow \mathbb{R}$ is a function. What does it mean that $g$ is continuous on $\mathbb{R}$? Your answer should be a rigorous mathematical definition.)
(c) (4 points) The function \( h: \mathbb{R} \rightarrow \mathbb{R} \) is defined by

\[
h(x) = \begin{cases} 
  Cx & \text{if } x < 3, \\
  -x^2 & \text{if } x \geq 3,
\end{cases}
\]

where \( C \) is a constant (in other words, \( C \) is a fixed real number). Find all values of \( C \) that make \( h \) continuous on \( \mathbb{R} \).
3. Consider \( f(x) = \frac{-x^2 + 100}{x^2 - 2x + 1} \).

(a) (2 points) Find all numbers \( c \) such that \( f \) is continuous at \( x = c \).
3. (continued) Consider \( f(x) = \frac{-x^2 + 100}{x^2 - 2x + 1} \).

(b) (3 points) Decide whether \( \lim_{x \to \infty} f(x) \) exists. If it exists, compute it.

(c) (3 points) Decide whether \( \lim_{x \to -\infty} f(x) \) exists. If it exists, compute it.
3. (continued) Consider \( f(x) = \frac{-x^2 + 100}{x^2 - 2x + 1} \).

(d) (2 points) Find an equation of each horizontal asymptote of the graph of \( f \). If there are none, say so and explain why.
4. (16 points) Prove that the equation \( \frac{x^2}{x^7+1} - 0.44411 = 0 \) has at least one solution.
5. For each of the following limits, either evaluate or state why the limit does not exist. Show and justify all steps.

(a) (4 points)
\[
\lim_{x \to 0} \frac{\sin(5\pi x)}{5\pi x}
\]

(b) (6 points)
\[
\lim_{x \to 0} \frac{\tan(2x)}{\sin(5\pi x)}
\]
5. (continued) For the following limit, either evaluate or state why the limit does not exist. Show and justify all steps.

(c) (10 points)

\[
\lim_{x \to 0} 2x^2 \sin \left( \frac{1}{x} \right)
\]
6. (25 points) Assume that

\[
\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = M,
\]

where \( c, L, \) and \( M \) are real numbers. Prove that

\[
\lim_{x \to c} (f(x) + g(x)) = L + M.
\]
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