The First Midterm

The first midterm will consist of six problems for a total of 100 points. Problem 6 on the first midterm will be chosen from the following list (up to a change in notation):

1. Assume that $\lim_{x \to c} f(x) = L$, where $c$ and $L$ are real numbers. Let $a$ be any real number. Prove that

$$\lim_{x \to c} af(x) = aL.$$

2. Assume that

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = M,$$

where $c, L,$ and $M$ are real numbers. Prove that

$$\lim_{x \to c} (f(x) + g(x)) = L + M.$$

[see p114-115]

3. Assume that $f$ and $g$ are differentiable. Prove that $f + g$ is differentiable and that $(f + g)' = f' + g'$.

[see p132]

4. Assume that $f$ is differentiable at $x = c$. Prove that $f$ is continuous at $x = c$.[see p136]

5. Assume that $f$ and $g$ are differentiable. Prove that $fg$ is differentiable and prove the product rule.

[see p144]

6. Prove that the functions sin and cos are differentiable and obtain – with proofs – their derivatives.

[see p165]

7. Compute with proof $\lim_{\theta \to 0} \frac{\sin \theta}{\theta}$.

Remember to prove all your statements.

Hint: Consider the circle of radius 1 centered at the origin. Let $B$ be a point on this circle lying in the first quadrant such that $B$ has coordinates $(\cos \theta, \sin \theta)$. Let $A$ be the point on this circle which lies on the positive $x$-axis. Finally, let $C$ denote the point of intersection between the line perpendicular to the $x$-axis at $A$ and the line passing through $O$ and $B$. Compare the areas of the triangle $OAB$, the sector of circle determined by $BOA$, and the triangle $COA$. [see p97]