Due on Wednesday, March 6, at 5:00pm

Your assignment should be written neatly and stapled, and have your full name written in capital letters on the front page. Assignments that fail to satisfy these conditions may be disregarded.

Any statement should be supported by a proof or by a clear citation of a theorem/definition-proof in the textbook. All steps of a computation should be clearly indicated and justified.

Writeups must be individual. If you have received help with solving a problem, then briefly cite your source.

Hand in:

1. (6pts) Let $U$ be an open subset of $\mathbb{C}$, $z_0 \in U$, and $f : U \to \mathbb{C}$ a function of class $C^1$ on $U$.

   Prove that $f$ satisfies the Cauchy-Riemann equations at $z_0$ if and only if $df_{z_0} : \mathbb{C} \to \mathbb{C}$ is $\mathbb{C}$-linear.

   Show that, in this case,
   $$df_{z_0}(z) = \frac{\partial f}{\partial z}(z_0)z = f'(z_0)z \quad \forall z \in \mathbb{C}.$$  

   Note: See also Problem F on homework 4.

2. (6pts) [KG, Chapter 2, Exercise 3] reworded

   Let $U \subseteq \mathbb{C}$ be an open disc with center 0. Let $f$ be a $\mathbb{C}$-differentiable function on $U$. If $z \in U$, then define $\gamma_z$ to be the path
   $$\gamma_z(t) \equiv tz \quad 0 \leq t \leq 1.$$

   Define
   $$F(z) = \int_{\gamma_z} f(\zeta)d\zeta.$$

   Prove that $F$ is a complex antiderivative for $f$.

   Note: You may not quote a proof in the textbook.

3. (6pts) [KG, Chapter 2, Exercise 43] reworded

   If $f$ is a $\mathbb{C}$-differentiable polynomial and if
   $$\int_{\partial D_1(0)} f(z)\bar{\zeta}dz = 0 \quad \forall j \in \mathbb{Z}^\geq 0,$$

   then prove that $f = 0$.

4. (4pts) [Pal, Chapter 5, Section 8, Problem 8.16] An entire function $f = u + iv$ has the feature that
   $$u_xv_y - u_yv_x = 1$$

   throughout the complex plane. Demonstrate that $f$ has the form $f(z) = az + b$, where $a, b$ are constants and $|a| = 1$.

   Note: In the above problem, $u$ and $v$ are real-valued.

2.3: 4 (4pts), 6 (4pts)

\[1\] except for the case when the help was provided by the course instructor
Solve, but do not hand in:

2.3: 1, 3, 5, 7, 13, 14

*It is your duty to make sure that you understand why points were taken off your homework and what the correct solution in each case is. You should therefore analyze your graded assignments carefully and ask questions (during office hours and/or whenever invited to do so during the lectures).*

**References**
