Due on Wednesday, April 10, at 5:00pm

Your assignment should be written neatly and stapled, and have your full name written in capital letters on the front page. Assignments that fail to satisfy these conditions may be disregarded.

Any statement should be supported by a proof or by a clear citation of a theorem/definition/proof in the textbook. All steps of a computation should be clearly indicated and justified.

Writeups must be individual. If you have received help with solving a problem, then briefly cite your source.¹

Hand in:

(1) Let \( U \) and \( V \) be open subsets of \( \mathbb{C} \).
   
   (a) (2pts) Let \( g: U \rightarrow V \) and \( f: V \rightarrow \mathbb{C} \) be functions of class \( C^1 \). Derive Chain Rule formulas for
   
   \[ \frac{\partial(f \circ g)}{\partial z} \quad \text{and} \quad \frac{\partial(f \circ g)}{\partial \bar{z}} \]
   
   in terms of the operators \( \frac{\partial}{\partial z} \) and \( \frac{\partial}{\partial \bar{z}} \) only.
   
   (b) (2pts) Let \( u: U \rightarrow \mathbb{R} \) be a function of class \( C^2 \). Derive a formula for \( \Delta u \) in terms of the operators \( \frac{\partial}{\partial z} \) and \( \frac{\partial}{\partial \bar{z}} \) only.
   
   (c) (2pts) Assume that \( f \) and \( g \) are as in part (a) above, that \( f \) is of class \( C^2 \), and that \( g \) is \( \mathbb{C} \)-differentiable. Derive a formula for \( \Delta(f \circ g) \) in terms of \( \Delta f \) and \( g' \).
   
   (d) (2pts) Assume that \( f \) and \( g \) are as in part (c) above and that \( f \) is a real-valued harmonic function. Prove that \( f \circ g \) is harmonic.

(2) (2pts) Show that the series
   
   \[ \sum_{n=1}^{\infty} n^{-z} \]
   
   is absolutely convergent in \( \{ z \in \mathbb{C} : \Re z > 1 \} \).

(3) [KG, Chapter 3, Exercise 44] (4pts) If \( f: D(0,1) \rightarrow \mathbb{C} \) is a function, \( f^2 \) is holomorphic, and \( f^3 \) is holomorphic, then prove that \( f \) is holomorphic.

   My note: Holomorphic means \( \mathbb{C} \)-differentiable. \( D(0,1) \) is the open disc of center 0 and radius 1.

(4) 2.5: 8(4pts), 10(4pts), 12(4pts), 14(4pts)

Solve, but do not hand in:

2.5: 7, 9, 11, 15, 16, 19, 20, 24

It is your duty to make sure that you understand why points were taken off your homework and what the correct solution in each case is. You should therefore analyze your graded assignments carefully and ask questions (during office hours and/or whenever invited to do so during the lectures).

References


¹except for the case when the help was provided by the course instructor