Your full name in capital letters:

ID number:

Midterm Exam # 2

DO NOT OPEN THIS EXAM OR BEGIN WRITING until the course instructor has announced the beginning of the examination!

- No calculators. No cell phones. No questions. No textbooks or additional materials.
- Any statement should be supported by a proof or by a clear citation of a theorem/definition. All steps of a computation should be clearly indicated and justified. Answers without justification may receive zero credit.
- Hand in the exam no later than 6:20pm.
- Use blue or black ballpoint pens please. Answers written in pencil will be disregarded.
- Write neatly. Show answers and arguments in the space provided. You may use the back of the pages also, but indicate clearly any such material that you want marked.
- Incorrect statements will be penalized. If you do not want something graded, please cross it out.

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1. (3 points) Let

\[ f(z) = e^{\frac{17}{z^7}} \frac{z^3 + 7}{z^7(z^2 + 4)^3}. \]

Let \( C \) be the disjoint union of the piecewise \( C^1 \), simple, closed curves \( C_0, C_1, C_2 \) in the figure below, oriented as in the figure below.

Compute \( \int_C f(z) \, dz \).

Note: A piecewise \( C^1 \) curve is called piecewise smooth in the textbook.
Blank page in case you need more space for problem 1.
2. (4 points) Integrate the function
\[ \frac{z}{z^2 - 1} \]
along the circle \(|z| = 3\) in the positive sense.
Blank page in case you need more space for problem 2.
3. (4 points) Let $\gamma: [a, b] \longrightarrow \mathbb{C}$ be a curve of class $C^1$. Let $f: D \longrightarrow \mathbb{C}$ be a $C$-differentiable function on the open set $D$, $D \subseteq \mathbb{C}$. Assume that
\[ \gamma([a, b]) \subseteq D. \]
Prove that
\[ (f \circ \gamma)'(t) = f'(\gamma(t))\gamma'(t) \quad \forall t \in [a, b]. \]

*Note:* The fact that $\gamma: [a, b] \longrightarrow \mathbb{C}$ is a curve of class $C^1$ means that $\gamma'$ exists and is continuous on $[a, b]$. 
4. (2 points) Assume that $\gamma : [a, b] \rightarrow \mathbb{C}$ is a piecewise $C^1$ curve. Let $f : D \rightarrow \mathbb{C}$ be a $C^1$-differentiable function on the open set $D$, $D \subseteq \mathbb{C}$. Assume that

$$\gamma([a, b]) \subseteq D.$$ 

Prove that

$$\int_{\gamma} f'(z)dz = f(\text{endpoint}) - f(\text{initial point}),$$

where

initial point $\equiv \gamma(a)$ and endpoint $\equiv \gamma(b)$. 


Blank page in case you need more space for problem 4.
5. (4 points) Let \( r \) and \( R \) be positive numbers such that \( r < R \).

Is there a \( C \)-differentiable function \( F \) on the annulus

\[ A \equiv \{ z \in \mathbb{C} : r < |z| < R \} \]

such that \( F'(z) = \frac{1}{z} \) on \( A \)?
6. (4 points) Let $\gamma$ be any piecewise $C^1$ curve in the domain $\{z \in \mathbb{C} : \text{Im} z > 0\}$ which joins $-1 + 2i$ to $1 + 2i$.

Evaluate the integral

$$\int_{\gamma} \frac{z}{z + 1} \, dz.$$
Blank page in case you need more space for problem 6.
7. (4 points) Assume that $f$ is an entire function and $\text{Re} f(z) \leq c$ for all $z \in \mathbb{C}$. Prove that $f$ is constant.

*Note:* Entire means $\mathbb{C}$-differentiable on $\mathbb{C}$.

*Hint:* Consider $e^{f(z)}$. 