Terminology/Conventions

• The symbol $f: A \to B$ will be taken to mean that $f$ is a function defined on the set $A$ and with values in the set $B$.

• Let $U \subseteq \mathbb{C}$ be an open set and $z_0 \in U$. Let $f: U \to \mathbb{C}$. We say that $f$ is $\mathbb{C}$-differentiable (or complex differentiable) at $z_0$ if and only if the limit

$$\lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

exists in $\mathbb{C}$. If $f$ is $\mathbb{C}$-differentiable at $z_0$, we denote by $f'(z_0)$ the limit in (1) and we call $f'(z_0)$ the complex derivative (or simply the derivative) of $f$ at $z_0$.

• Let $U \subseteq \mathbb{C}$ be an open set and $z_0 \in U$. Let $f: U \to \mathbb{C}$. We say that $f$ is analytic at $z_0$ if and only if there exists $r > 0$ and a sequence of complex numbers $(a_n)_{n \geq 0}$ such that

$$f(z) = \sum_{n=0}^{\infty} a_n(z - z_0)^n \text{ for all } z \in U \text{ with } |z - z_0| < r.$$